ADVANCED

General Certificate of Education

2011

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

FRIDAY 24 JUNE, AFTERNOON

MARK SCHEME

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M   indicates marks for correct method.

W   indicates marks for working.

MW   indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a)   a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b)   readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1   Centre of circle is (4, 7)   MW1

Gradient of radius $= \dfrac{4-7}{8-4}$ M1

$= -\dfrac{3}{4}$ W1

Hence gradient of tangent is $\dfrac{4}{3}$ MW1

Therefore, equation of tangent is $(y-4) = \dfrac{4}{3}(x-8)$ M1

Tidying equation gives $y = \dfrac{4}{3}x - \dfrac{20}{3}$ W1

and hence $3y = 4x - 20$ 6

2   **(i)**

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 10 & 8 \\ 4 & -2 \end{pmatrix}$

Hence $3a + 4b = 10$ ① $3c + 4d = 4$ ③   M1

$5a - 2b = 8$ ② $5c - 2d = -2$ ④   M1

$2 \times$ ② gives $10a - 4b = 16$ ⑤ $2 \times$ ④ gives $10c - 4d = -4$ ⑥

① + ⑤ gives $13a = 26$ ③ + ⑥ gives $13c = 0$

Hence $a = 2$ Hence $c = 0$ W1

Hence $b = 1$ Hence $d = 1$ W1

This gives $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$

(ii) $\mathbf{S} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$ M1 M1

$= \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ W1

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(iii) For invariant lines $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} t \\ mt \end{pmatrix}$ M1

Expand to give $2mx = t$ MW1

$$2x = mt$$

Dividing gives $\dfrac{2m}{2} = \dfrac{1}{m}$ M1

Hence $2m^2 = 2$ W1

giving $m^2 = 1$

i.e. $m = \pm 1$ W1

Hence the invariant lines are $y = \pm x$ W1   13

3   <u>Closure:</u> $\begin{pmatrix} p & q \\ 3q & -p \end{pmatrix} + \begin{pmatrix} r & s \\ 3s & -r \end{pmatrix}$ M1M1

$$= \begin{pmatrix} p + r & q + s \\ 3q + 3s & -p - r \end{pmatrix}$$ W1

$$= \begin{pmatrix} p + r & q + s \\ 3(q + s) & -(p + r) \end{pmatrix}$$

Since this is of the same form as the original matrix, then closure holds.   M1

<u>Identity:</u> $\begin{pmatrix} p & q \\ 3q & -p \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} p & q \\ 3q & -p \end{pmatrix}$ M1

Since $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a member of **S**, with $p = q = 0$, then it is the identity matrix   W1

<u>Inverse:</u> $\begin{pmatrix} p & q \\ 3q & -p \end{pmatrix} + \begin{pmatrix} -p & -q \\ -3q & p \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ M1

Since $\begin{pmatrix} -p & -q \\ 3(-q) & p \end{pmatrix}$ is a member of **S**, then it is the inverse of $\begin{pmatrix} p & q \\ 3q & -p \end{pmatrix}$ W1

Since closure holds and an identity and inverse exist and we can assume the associative law, then **S** forms a group under matrix addition.   MW1   9

4 (i) If 3 is an eigenvalue then $\begin{pmatrix} -1 & p & 0 \\ p & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ M1

Hence $-x + py = 3x$ ①   W1

$px + 2z = 3y$ ②

$2y + z = 3z$ ③

③ $\Rightarrow y = z$ M1

② $\Rightarrow px + 2y = 3y$

Hence $px = y$

① $\Rightarrow -x + p^2x = 3x$ W1

$\Rightarrow p^2x = 4x$

$\Rightarrow p^2 = 4$ W1

Hence $p = \pm 2$ W1

(ii) $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ M1

Hence $\begin{vmatrix} -1 -\lambda & 2 & 0 \\ 2 & -\lambda & 2 \\ 0 & 2 & 1 -\lambda \end{vmatrix} = 0$ M1

Expand to give

$(-1 - \lambda)\,[-\lambda\,(1 - \lambda) - 4] - 2\,[2(1 - \lambda)] = 0$ M1

$\Rightarrow -(1 + \lambda)\,(\lambda^2 - \lambda - 4) - 4\,(1 - \lambda) = 0$

$\Rightarrow \lambda^3 + \lambda^2 - \lambda^2 - \lambda - 4\lambda - 4 + 4 - 4\lambda = 0$

$\Rightarrow \lambda^3 - 9\lambda = 0$ W1

$\Rightarrow \lambda\,(\lambda^2 - 9) = 0$

Hence the eigenvalues are $\lambda = 0, \pm 3$ W2

(iii)

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

M1

Hence $-x + 2y = 3x$ ①

M1

$2x + 2z = 3y$ ②

$2y + z = 3z$ ③

③ $\Rightarrow y = z$

① $\Rightarrow y = 2x$

Hence an eigenvector is $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

W2

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5 **(i)** For the solution not to be unique then $\begin{vmatrix} 3 & \lambda & -1 \\ 2\lambda & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = 0$ M1

$\Rightarrow 3(1) - \lambda(2\lambda) - 1(-2\lambda - 1) = 0$ M1

$\Rightarrow 3 - 2\lambda^2 + 2\lambda + 1 = 0$ W1

$\Rightarrow 2\lambda^2 - 2\lambda - 4 = 0$

$\Rightarrow \lambda^2 - \lambda - 2 = 0$ W1

$\Rightarrow (\lambda - 2)(\lambda + 1) = 0$

$\Rightarrow \lambda = 2, -1$ W2

(ii) When $\lambda = 2$ the equations become   M1

    $3x + 2y - z = 3$ ①

    $4x + y = 1$ ②

    $x - y + z = -2$ ③

① + ③ $\Rightarrow 4x + y = 1$ which is the same as ②   W1

Hence $y = 1 - 4x$

③ $\Rightarrow x - 1 + 4x + z = -2$

$\Rightarrow 5x + 1 + z = 0$

$\Rightarrow z = -1 - 5x$ W1

Hence the general solution is $(t, 1 - 4t, -1 - 5t)$ W1

When $\lambda = -1$ the equations become   M1

    $3x - y - z = 3$ ①

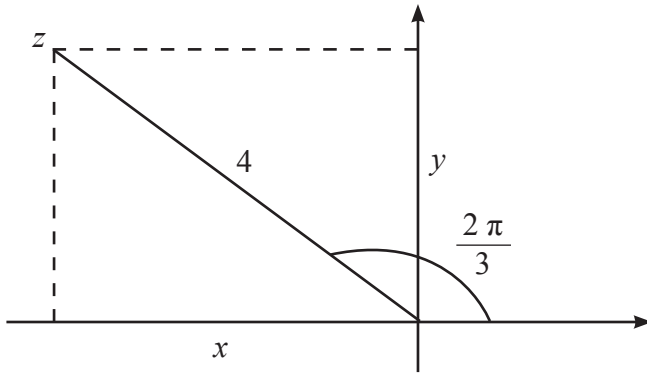 $-2x + y = 1$ ②

    $x - y + z = -2$ ③

① + ③ $\Rightarrow 4x - 2y = 1$ W1

$\Rightarrow -2x + y = -\dfrac{1}{2}$

which contradicts ②

Hence there is no solution   W1   13

6 (a)

M1

$x = 4 \cos \dfrac{2\pi}{3}$ $y = 4 \sin \dfrac{2\pi}{3}$ W2

Hence $z = -2 + 2\sqrt{3}\ i$ W1

(b) $\dfrac{3 - 4i}{2 + i} \times \dfrac{2 - i}{2 - i}$ M1

$\Rightarrow \dfrac{6 - 8i - 3i - 4}{4 + 1}$ W2

$\Rightarrow \dfrac{2 - 11i}{5}$

$\Rightarrow \dfrac{2}{5} - \dfrac{11}{5}\ i$ W2
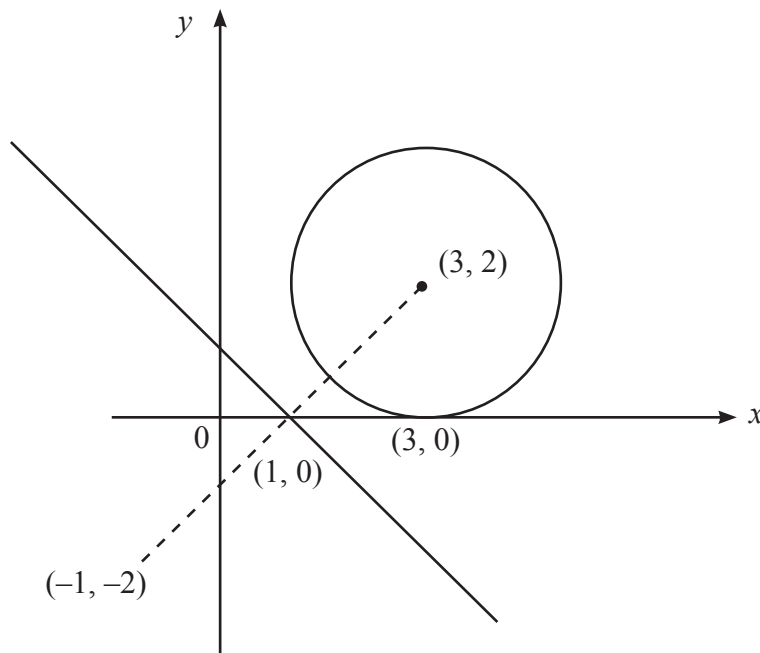
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(b) (i) Circle, centre (3,2), radius 2

See diagram in (ii)   MW3

(ii)

Perpendicular line though the midpoint of the line joining (3, 2) and (- 1, -2) i.e. (1, 0)   MW3

(iii) Distance from (1, 0) to (3, 2) is $\sqrt{4+4}$

$$= \sqrt{8}$$ MW1

This distance is greater than the radius of the circle i.e. 2   W1

Therefore, the line lies outside the circle and the two loci do not meet.   W1   18

Total   **75**