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**ADVANCED**
**General Certificate of Education**
**2011**

# Mathematics

## Assessment Unit M4

*assessing*

## Module M4: Mechanics 4

**[AMM41]**

AMM41

**WEDNESDAY 22 JUNE, MORNING**

**TIME**

1 hour 30 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

**INFORMATION FOR CANDIDATES**

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

**Answer all six questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** A uniform triangular lamina OAB is bounded by the $x$-axis, the line $x = a$ and the line $y = kx$. O is the origin and A is the point $(a, 0)$. The centre of mass of the lamina is G as shown in **Fig. 1** below.

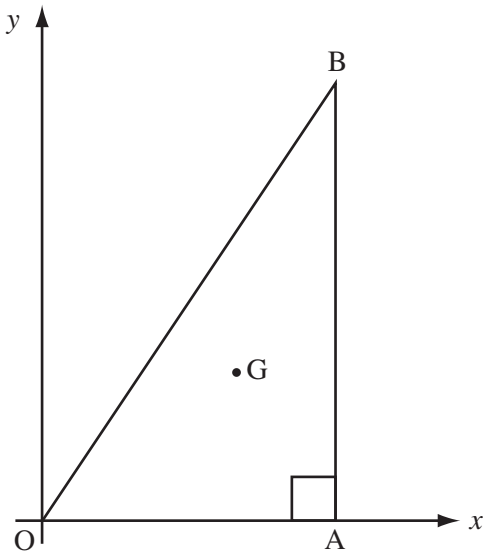


**Fig. 1**

The mass per unit area of the lamina is $\sigma$.

**(i)** Find in terms of $a$, $k$, and $\sigma$, the mass of the lamina and by using integration, its moments about the $x$- and $y$-axes. [8]

**(ii)** Find the coordinates of G. [2]

**(iii)** Verify that for this triangular lamina the coordinates of G can be found by summing the coordinates of the three vertices and dividing by three. [2]

**2** The star-ship FANTASIA has a probon beam engine. The zoomeration, Z, of the star-ship is believed to depend on the powel, $P$, struk, $S$ and dinesty, $D$, of the beam.
The dimensions of these quantities are shown in the table below.

| Quantity | Dimension |
|---|---|
| $Z$ | $[L^3][T^{-4}]$ |
| $P$ | $[M][L^{-2}][T]$ |
| $S$ | $[M^2][T^{-2}]$ |
| $D$ | $[M^{-1}][L^3][T^{-1}]$ |

Boris Blastov conjectures that

$$Z = kP^aS^bD^c$$

where $k$ is a dimensionless constant.

**(i)** Use the method of dimensions to find $a$, $b$ and $c$. [8]

During a trial of the engine the following measurements were made in S.I. units:

$$Z = 1.2 \times 10^{11},\ P = 2 \times 10^2,\ S = 3 \times 10^9,\ D = 5 \times 10^3$$

**(ii)** Show that $k = 1.6 \times 10^{12}$ [3]

During further trials, when $P$ was doubled $Z$ increased by a factor of 5

**(iii)** State whether or not Boris's conjecture could be correct. Briefly explain your answer. [2]

**3** Three spheres A, B, and C of equal radius and masses $4m$, $2m$ and $m$ respectively are at rest in a straight line on a smooth horizontal surface. The coefficient of restitution between each pair of spheres is 0.5
A is projected towards B with speed $u$ and collides directly with it.
After the collision A and B move off in the same direction, B with speed $w$ and A with speed $v$.

**(i)** Show that $w = u$ and find $v$. [6]

A collision now occurs between B and C.

**(ii)** Find the speeds of B and C after this collision. [4]

**(iii)** State if any further collisions will occur, briefly explaining your answer. [1]

**4** In a distant galaxy, the planet Fido has two moons, Ria and Via.
The masses of the planet and its moons are $M$, $m_1$ and $m_2$ respectively.
The orbit of Ria has a radius $r$ and the coplanar orbit of Via has a radius $2.08r$
The universal gravitational constant is G.
Ria moves around its orbit at speed $v_1$ and Via moves at $v_2$

**(i)** Show that $v_1 = 1.44v_2$ approximately. [5]

The angular velocities of Ria and Via are $\omega_1$ and $\omega_2$ respectively.

**(ii)** By using the result in **(i)**, or otherwise, show that $\omega_1 = 3\omega_2$ approximately. [3]

The period of Via is 63 days.

**(iii)** Find the period of Ria. [3]

**5** The "Dip" is part of a competitive cycle trail where competitors must keep their bikes in contact with the track at all times.
**Fig. 2** below shows the vertical section along the "Dip".

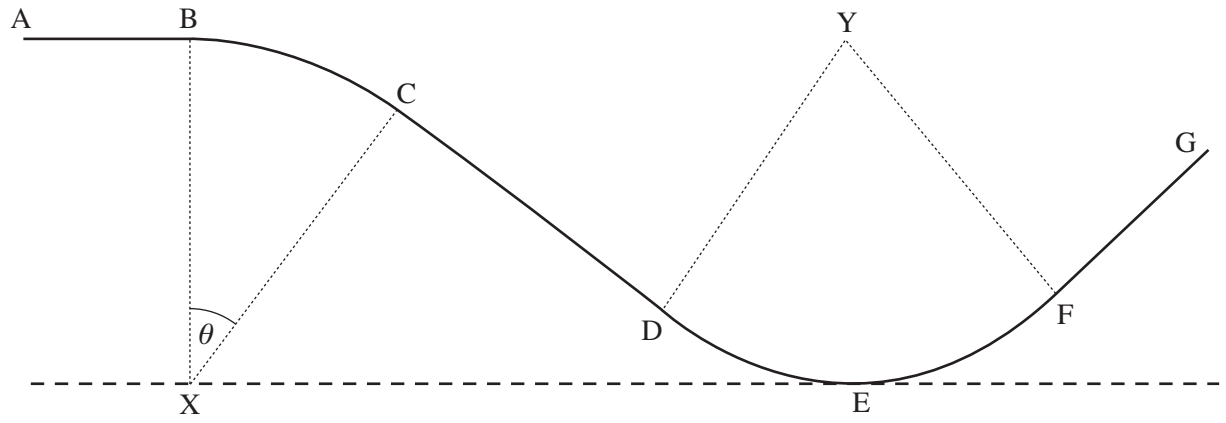


**Fig. 2**

There is a horizontal approach AB, a circular arc BC, centre X, a straight incline CD and another circular arc DEF, centre Y, followed by an upward incline FG.
Both arcs are of radius 5 m. BC subtends $\theta°$ at its centre X where $\cos\theta = 0.8$
A, B and Y are at the same horizontal level.
E is 5 m vertically below the level of A and B.
Decla approaches the "Dip" along AB reaching B with a speed of $4\,\text{m}\,\text{s}^{-1}$
Model Decla and her bike as a particle of mass 70 kg.

**(i)** Find the force exerted by the track on the bike and rider at E. [7]

At C, Decla and her bike experience a normal reaction, $N$ newtons, from the track.

**(ii)** Find $N$. [6]

**(iii)** Explain briefly why the bike remains in contact with the track over the arc BC. [2]

**(iv)** State what further modelling assumption you have made in answering this question. [1]

**6** The triangle ABC has sides:

$$AB = 0.6\,m$$
$$BC = 0.8\,m$$
$$AC = 1.0\,m$$

D is the point on AC such that BD is perpendicular to AC.
Forces of 12 N, 9 N and 8 N act along the sides BA, BC and AC respectively as shown in **Fig. 3** below.

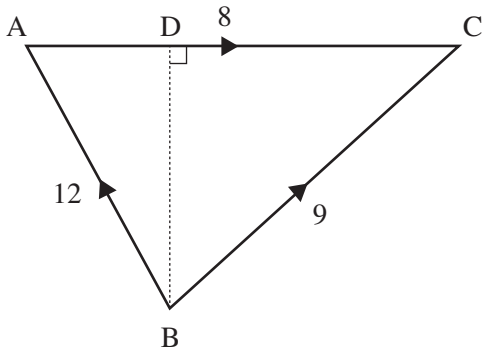


**Fig. 3**

The resultant of these forces is *R*.

**(i)** Show that *R* passes through D and find its magnitude. [7]

The forces are now replaced and 3 N acts along BA, 5 N along AC and 4 N along CB as shown in **Fig. 4** below.

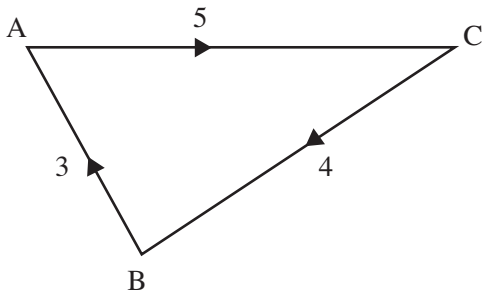


**Fig. 4**

**(ii)** Show that this system reduces to a couple and find its moment. [5]

**THIS IS THE END OF THE QUESTION PAPER**