Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

TUESDAY 31 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ell n\, z$ where it is noted that $\ell n\, z \equiv \log_e z$

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Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find, in radians, the general solution of the equation

$$2\sec^2\theta - 3\tan\theta - 1 = 0$$ [6]

2 If $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$, prove by mathematical induction that

$$\mathbf{A}^n = \begin{pmatrix} 1 & 0 \\ 5n & 1 \end{pmatrix}$$

where n is a positive integer. [5]

3 **(i)** Find the sum of the series

$$\frac{1}{1\times 4} + \frac{1}{2\times 5} + \frac{1}{3\times 6} + \ldots + \frac{1}{n(n+3)}$$ [8]

(ii) Hence find

$$\sum_{r=1}^{\infty} \frac{1}{r(r+3)}$$ [1]

4 **(i)** Find the equation of the parabola with focus (2, 2) and directrix $x = 8$ [7]

The latus rectum of a parabola is the chord parallel to the directrix through the focus.

(ii) Find the length of the latus rectum of the parabola in part **(i)**. [3]

5 Solve the differential equation

$$\frac{dy}{dx} + y\cot x = \cos^3 x$$

given that $y = \frac{3}{8\sqrt{2}}$ when $x = \frac{\pi}{4}$ [10]

6 **(i)** Use Maclaurin's theorem to find the first four terms in the expansion of

$$\frac{1}{1+x}$$

where $|x| < 1$ [4]

(ii) Write

$$\frac{2x^2 + x + 27}{(9 + x^2)(1 - x)}$$

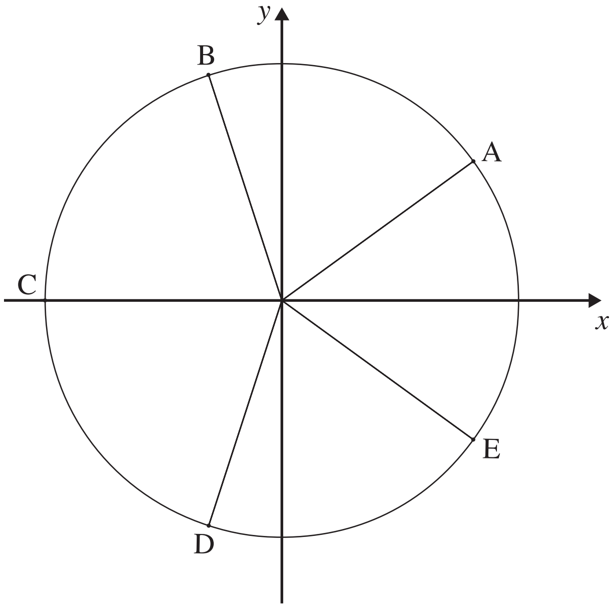
in partial fractions. [6]

(iii) Hence, or otherwise, derive the first four terms in the expansion of

$$\frac{2x^2 + x + 27}{(9 + x^2)(1 - x)}$$

[6]

7 **(i)** Find, in the form $re^{i\theta}$, the values of the 5 roots of the equation $z^5 + 32 = 0$, which are shown in **Fig. 1** below.



[6]

Fig. 1

(ii) Show that a quadratic equation whose roots are A and E is given by

$$z^2 - 4z\cos\frac{\pi}{5} + 4 = 0$$ [4]

A quadratic equation whose roots are B and D is given by

$$z^2 + 4z\cos\frac{2\pi}{5} + 4 = 0$$

(iii) Explain why

$$(z+2)\left(z^2 + 4z\cos\frac{2\pi}{5} + 4\right)\left(z^2 - 4z\cos\frac{\pi}{5} + 4\right) = 0$$

is an equation with roots A, B, C, D, E. [1]

(iv) By comparing the coefficients of z^4 in the equations in parts **(iii)** and **(i)** show that

$$\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$$ [8]