



Rewarding Learning

ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

FRIDAY 28 JANUARY, MORNING

**MARK
SCHEME**

1 (a) (i) $\frac{1}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$ MW1

$1 = A(x-1) + B(x+1)$ MW1

Let $x = 1$ $1 = 2B \Rightarrow B = \frac{1}{2}$ MW1

$x = -1$ $1 = 2A \Rightarrow A = -\frac{1}{2}$ W1

(ii) $\frac{1}{2} \int \frac{1}{(x-1)} - \frac{1}{(x+1)} dx$ MW1

$= \frac{1}{2} \ln |x-1| - \frac{1}{2} \ln |x+1| + c$ MW3

(b) $\int x \cos x dx$

$= x \sin x - \int \sin x dx$ M1W2

$= x \sin x + \cos x + c$ W2

2 Using dot product

$$\mathbf{p} \cdot \mathbf{q} = |\mathbf{p}| |\mathbf{q}| \cos \theta \quad \text{M1}$$

$$(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \cdot (4\mathbf{i} + 5\mathbf{j} + \mathbf{k}) \quad \text{M1}$$

$$= 4 - 5 - 2 = -3 \quad \text{W1}$$

$$|\mathbf{i} - \mathbf{j} - 2\mathbf{k}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6} \quad \text{M1W1}$$

$$|4\mathbf{i} + 5\mathbf{j} + \mathbf{k}| = \sqrt{4^2 + 5^2 + 1^2} = \sqrt{42} \quad \text{MW1}$$

$$-3 = \sqrt{6} \sqrt{42} \cos \theta$$

$$\frac{-3}{\sqrt{252}} = \cos \theta$$

$$\theta = 100.89^\circ \quad \text{W1}$$

(101°)

7

$$3 \quad \cos x = 2 [\sin x \cos 60^\circ + \cos x \sin 60^\circ] \quad \text{M1W1}$$

$$\cos x = 2 \left[\sin x \frac{1}{2} + \cos x \frac{\sqrt{3}}{2} \right] \quad \text{MW1}$$

$$\cos x = \sin x + \sqrt{3} \cos x$$

$$(1 - \sqrt{3}) \cos x = \sin x$$

$$1 - \sqrt{3} = \tan x \quad \text{M1W1}$$

$$x = -36.2^\circ \text{ or } 144^\circ \quad \text{MW2}$$

7

4	(a) (i) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$	M1	9
	$1 + x^2 = y^2$	W1	
	(ii) $y^2 = 1 + x^2$		
	$2y \frac{dy}{dx} = 2x$	M2W2	
	$\frac{dy}{dx} = \frac{x}{y}$		
	$y^2 = 1 + 1$		
	$= 2$		
	$y = \pm \sqrt{2}$	MW2	
	$\frac{dy}{dx} = \frac{1}{\pm \sqrt{2}}$	MW1	
5	(i) LHS $\tan x \sec^4 x = \tan x \sec^2 x \sec^2 x$	MW1	10
	$= \tan x (1 + \tan^2 x) \sec^2 x$	M1	
	$= \tan x \sec^2 x + \tan^3 x \sec^2 x$	W1	
	$= \text{RHS}$		
	(ii) $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x + \tan^3 x \sec^2 x \, dx$	MW1	
	$u = \tan x \quad \frac{du}{dx} = \sec^2 x$	MW1	
	$\int_0^{\frac{\pi}{4}} (\tan x + \tan^3 x) \sec^2 x \, dx$		
	$\int (u + u^3) \sec^2 x \frac{du}{\sec^2 x}$	M1W1	
	$= \left[\frac{u^2}{2} + \frac{u^4}{4} \right]$	MW1	
	$= \left[\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} \right]_0^{\frac{\pi}{4}}$	M1	
	$= \frac{3}{4}$	MW1	

6 $(1+x^2) \frac{dy}{dx} = x(1+y)$

$$\int \frac{dy}{1+y} = \int \frac{x dx}{(1+x^2)}$$

M2W1

$$\ln |1+y| = \frac{1}{2} \ln |1+x^2| + c$$

W2

$$x = 0 \text{ when } y = 0 \therefore c = 0$$

M1W1

$$\ln |1+y| = \frac{1}{2} \ln |1+x^2|$$

$$1+y = \sqrt{1+x^2}$$

M2W1

$$y = \sqrt{1+x^2} - 1$$

W1

11

7 Volume = $\int_0^9 \pi y^2 dx$

M2W1

$$= \pi \int_0^9 e^{\frac{x}{3}} dx$$

W1

$$= \pi \left[3e^{\frac{x}{3}} dx \right]_0^9$$

MW1

$$= 3\pi(e^3 - 1)$$

W2

7

8 (i) $y = \sin x$

$x = \sin^{-1} y$

MW1

$f^{-1} : x \rightarrow \sin^{-1} x$

MW1

domain $-1 \leq x \leq 1$

W1

range $-\frac{\pi}{2} \leq f^{-1}(x) \leq \frac{\pi}{2}$

W1

(ii) $x \rightarrow \sin x \rightarrow |\sin x|$

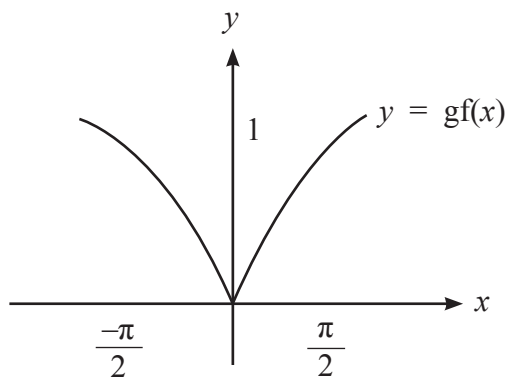
M1W1

$gf : x \rightarrow |\sin x|$

range $0 \leq gf(x) \leq 1$

W2

(iii)



MW3

11

Total

75