GCE A2

Mathematics

Summer 2010

Mark Schemes

Issued: October 2010

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE) AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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General Certificate of Education
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[AMC31]

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MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

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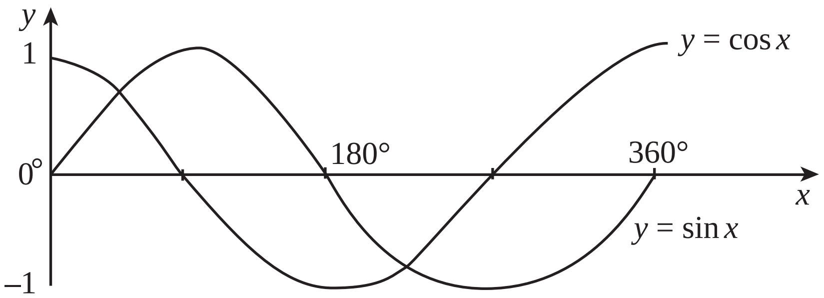
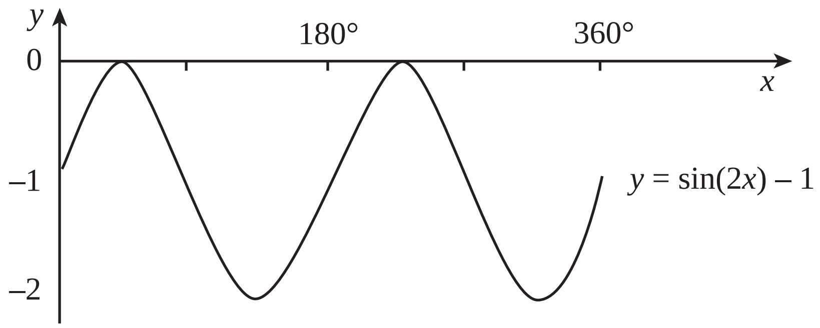
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	$5x + 3 < 2 \quad 5x + 3 > -2$	M1	
	$5x < -1 \quad 5x > -5$		
	$-1 < x < -\frac{1}{5}$	W3	4
2	**(i)** 	MW2	
	(ii) $\sin x = \cos(x - 90°)$ $a = 90°$	MW1	
	(iii) 	MW2	5
3	**(a)** $y = x^2 \ln x$		
	$u = x^2 \quad \frac{du}{dx} = 2x$		
	$v = \ln x \quad \frac{dv}{dx} = \frac{1}{x}$		
	$\frac{dy}{dx} = x^2 \frac{1}{x} + 2x \ln x$	M1	
	$= x + 2x \ln x$	W2	
	(b) $\int 3x^2 + e^{-x} - \operatorname{cosec} x \cot x + \frac{3}{x}\, dx$		
	$x^3 - e^{-x} + \operatorname{cosec} x + 3 \ln x + c$	MW5	8

			AVAILABLE MARKS
4	$x + 1 = \tan t$	MW1	
	$y - 1 = \cot^2 t$	MW1	
	$= \dfrac{1}{\tan^2 t}$	M1	
	$y - 1 = \dfrac{1}{(x+1)^2}$	M1W1	5
5	**(a)** $\dfrac{2x-7}{(x-3)^2} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2}$	M1W1	
	$2x - 7 = A(x-3) + B$	M1	
	Put $x = 3$ $\quad -1 = B$	M1W1	
	Compare coeff of x $\quad 2 = A$	MW1	
	$= \dfrac{2}{x-3} - \dfrac{1}{(x-3)^2}$		
	(b) $\dfrac{1}{(3-x)^2} = (3-x)^{-2}$	MW1	
	$= \dfrac{1}{9}\left[\left(1 - \dfrac{x}{3}\right)^{-2}\right]$	M1W1	
	$= \dfrac{1}{9}\left[1 + (-2)\left(-\dfrac{x}{3}\right) + \dfrac{(-2)(-3)}{2}\left(-\dfrac{x}{3}\right)^2 + \ldots\right]$	MW3	
	$= \dfrac{1}{9}\left[1 + \dfrac{2}{3}x + \dfrac{1}{3}x^2 + \ldots\right]$		
	$= \dfrac{1}{9} + \dfrac{2}{27}x + \dfrac{1}{27}x^2 + \ldots$	W1	13

			AVAILABLE MARKS
6	**(a)** $f(x) = 4e^{-x} - x$		
	$f'(x) = -4e^{-x} - 1$	M1W2	
	$x_0 = 1.3$		
	$x_1 = 1.3 - \frac{-0.20987}{-2.09013} = 1.1996$	M1W1	
	$x_2 = 1.1996 - \frac{-0.00478}{-2.20478} = 1.20$	W2	
	(b) **(i)** $\frac{1}{2}N_0 = N_0 e^{-k5730}$	M1	
	$\ln\frac{1}{2} = \ln e^{-5730k}$	M1	
	$k = \frac{\ln 2}{5730} = 0.000121$	MW1	
	(ii) $N_0 e^{-1000\left(\frac{\ln 2}{5730}\right)}$	MW1	
	$=$ 88.6	W1	12
7	**(a)** $4\sin x + 1 = 3\operatorname{cosec} x$		
	$4\sin^2 x + \sin x - 3 = 0$	M2W1	
	$(4\sin x - 3)(\sin x + 1) = 0$		
	$\sin x = \frac{3}{4}$ or $\sin x = -1$	MW1	
	$x = 48.6°, 131°$ or $270°$	MW3	
	(b) LHS $= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$	M1W2	
	$= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$	MW2	
	$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$		
	$= \tan\theta =$ RHS	W1	13

			AVAILABLE MARKS
8 **(i)**	To find area under curve		
	$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec^2 x \, dx$	M2W1	
	$= [\tan x]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$	MW1	
	$A = \sqrt{3} - (-\sqrt{3})$		
	$= 2\sqrt{3}$	MW1	
	Shaded reaa $= 2 \times \frac{\pi}{3} \times 4 - 2\sqrt{3}$	M1W1	
	$= \frac{8}{3}\pi - 2\sqrt{3}$	MW1	
(ii)	To find gradient of tangent		
	$y = \sec^2 x$		
	$\frac{dy}{dx} = 2 \sec x \sec x \tan x$	M1W2	
	$= 2 \sec^2 x \tan x$	MW1	
	$x = -\frac{\pi}{6}, m = 2 \times \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} \times \left(-\frac{1}{\sqrt{3}}\right) = \frac{-8}{3\sqrt{3}}$	MW1	
	$x = -\frac{\pi}{6}, y = \sec^2 x = \frac{4}{3}$	M1	
	$y - \frac{4}{3} = -\frac{8}{3\sqrt{3}}\left(x - \left(-\frac{\pi}{6}\right)\right)$		
	$y + \frac{8}{3\sqrt{3}}x = \frac{4}{3} - \frac{4\pi}{9\sqrt{3}}$		
	$(y + 1.54x = 0.527)$	W1	15
		Total	**75**

Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

MONDAY 24 MAY, AFTERNOON

MARK SCHEME

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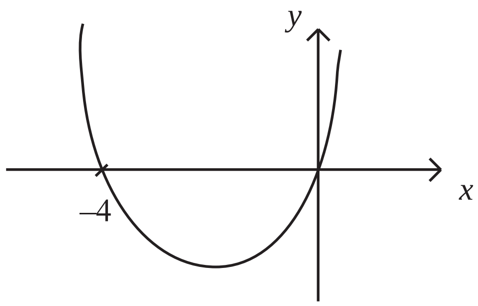
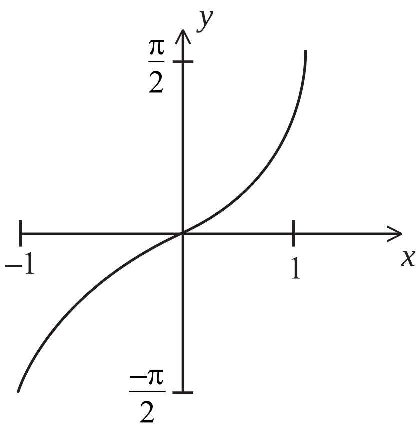
				AVAILABLE MARKS
1	**(i)**	$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP} = -\mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + 4\mathbf{i} + 4\mathbf{j}$	M1	
		$\overrightarrow{QP} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$	W1	
	(ii)	$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR} = -\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$	MW1	
	(iii)	$\overrightarrow{QP}.\overrightarrow{QR} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} . \begin{pmatrix} -1 \\ 6 \\ 3 \end{pmatrix}$	M1	
		$= -3 + 12 - 9 = 0$	MW1	
		$\therefore QP \perp QR$	MW1	6
2		$V = \int_0^4 \pi \left(2\sqrt{x} + 3\right)^2 dx$	M2	
		$V = \pi \int_0^4 4x + 12\sqrt{x} + 9\, dx$	MW1	
		$V = \pi \left[2x^2 + 8x^{\frac{3}{2}} + 9x \right]_0^4$	MW2	
		$V = 132\pi \approx 415 \text{ units}^2$	M1 W1	7

		AVAILABLE MARKS
3 (a) 	M1W1	
$a = -2$	MW1	
(b) (i) $b = 3$	MW1	
(ii) $y = \dfrac{4x}{x-3}$	M1	
$xy - 3y = 4x$		
$xy - 4y = 3y$		
$x\left(y-4\right) = 3y$		
$x = \dfrac{3y}{y-4}$	M1W1	
$g^{-1}\left(x\right) = \dfrac{3x}{x-4}$	MW1	
$\left\{x : x \in \mathbb{R}, x \neq 4\right\}$	MW1	
(iii) $g\left(x\right) \in \mathbb{R}, g\left(x\right) \neq 4$	MW1	10
4 (i) $\dfrac{dP}{dh} \propto P$	M1	
$\dfrac{dP}{dh} = kP$	W1	
(ii) $\int \dfrac{1}{p}\, dP = \int k\, dh$	M2W1	
$1\text{n}\, P = kh + c$	MW1	
$h = 0, P = 100\,000 \Rightarrow c = 1\text{n}\, 100\,000$	M1	
$h = 1, P = 88\,000 \Rightarrow 1\text{n}\, 88\,000 = k + 1\text{n}\, 100\,000$		
$k = -0.12783 \approx -0.128$	W1	
$1\text{n}\, P = -0.128h + 1\text{n}\, 100\,000$		
$h = 0.4 \Rightarrow 1\text{n}\, P = -0.128 \times 0.4 + 1\text{n}\, 100\,000$	M1	
$P = 95015.2 \text{ Pa} \approx 95\,000 \text{ Pa}$	W1	10

			AVAILABLE MARKS
5 **(a)**	$u = x - 2 \quad \frac{du}{dx} = 1$	MW1	
	$\int \frac{3x}{\sqrt{x-2}} dx = \int \frac{3u+6}{\sqrt{u}} du$	M1W1	
	$= \int 3u^{\frac{1}{2}} + 6u^{-\frac{1}{2}} du$	MW1	
	$= 2u^{\frac{3}{2}} + 12u^{\frac{1}{2}} + c$	MW2	
	$= 2(x-2)^{\frac{3}{2}} + 12(x-2)^{\frac{1}{2}} + c$	MW1	
	$= (x-2)^{\frac{1}{2}}(2x+8) + c$		
(b)	$u = 4x \quad \frac{dv}{dx} = \cos 2x$	M1	
	$\frac{du}{dx} = 4 \quad v = \frac{1}{2}\sin 2x$	W2	
	$I = \left[2x\sin 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 2\sin 2x\,dx$	M1W1	
	$I = \left[2x\sin 2x + \cos 2x\right]_0^{\frac{\pi}{4}}$	MW1	
	$I = \frac{\pi}{2} - 1$	MW1	14

			AVAILABLE MARKS
6	**(i)** $\dfrac{dx}{dt} = 4t^3 \quad \dfrac{dy}{dt} = 4t - 8$	MW2	
	$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$	M1	
	$\dfrac{dy}{dx} = \dfrac{4t-8}{4t^3} = \dfrac{t-2}{t^3}$	W1	
	(ii) $\dfrac{dy}{dx} = \dfrac{t-2}{t^3} = 0$	M1	
	$t = 2$	W1	
	$x = 10$ and $y = -2$	MW2	
	$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx}$	M2	
	$\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = -2t^{-3} + 6t^{-4}$		
	$\dfrac{d^2y}{dx^2} = \left(\dfrac{-2}{t^3} + \dfrac{6}{t^4}\right) \times \dfrac{1}{4t^3} = \dfrac{6-2t}{4t^7}$	W2	
	$t = 2 \quad \dfrac{d^2y}{dx^2} = \dfrac{2}{512} \Rightarrow +\text{ve} \;\therefore \text{min}$	MW1	13
	$(10, -2)$ is a min		

			AVAILABLE MARKS
7	**(a)** 	M1MW1	
	$-1 \leqslant x \leqslant 1$	MW1	
	(b) $2\sin\theta\cos\theta = \cos\theta$	MW1	
	$\cos\theta(2\sin\theta - 1) = 0$	MW2	
	$\cos\theta = 0 \quad \sin\theta = \frac{1}{2}$		
	$\theta = \pm\frac{\pi}{2} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$	MW2	
	(c) LHS $= \dfrac{1 + \dfrac{\sin^2 x}{\cos^2 x}}{1 - \dfrac{\sin^2 x}{\cos^2 x}}$	M1W1	
	$= \dfrac{\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$	M1W1	
	$= \dfrac{1}{\cos^2 x - \sin^2 x}$		
	$= \dfrac{1}{\cos 2x}$	M1W1	
	$= \sec 2x =$ RHS	MW1	15
		Total	**75**

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit F2
assessing
Module FP2: Further Pure Mathematics 2

[AMF21]

TUESDAY 22 JUNE, AFTERNOON

MARK SCHEME

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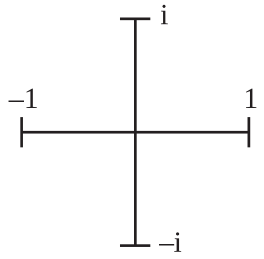
1	$z = \cos\theta + \mathrm{i}\sin\theta$		
	$z^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$	MW1	
	$z^3 = \cos^3\theta + 3\cos^2\theta\,\mathrm{i}\sin\theta$		
	$+ 3\cos\theta\,\mathrm{i}^2\sin^2\theta + \mathrm{i}^3\sin^3\theta$	MW1	
	Compare imaginary parts	M1	
	$\sin 3\theta = 3\cos^2\theta\sin\theta + (-1)\sin^3\theta$	W1	
	$= 3\left(1 - \sin^2\theta\right)\sin\theta - \sin^3\theta$		
	$= 3\sin\theta - 4\sin^3\theta$	W1	5
2	$\sum = \sum_{n+1}^{2n}\left(4k^3 - k\right)$	MW1	
	$= 4\sum_{n+1}^{2n} k^3 - \sum_{n+1}^{2n} k$	W1	
	$= \left(4\sum_{1}^{2n} k^3 - 4\sum_{1}^{n} k^3\right)$		
	$-\left(\sum_{1}^{2n} k - \sum_{1}^{n} k\right)$	M1	
	$= 4\cdot\frac{1}{4}(2n)^2(2n+1)^2 - 4\frac{1}{4}n^2(n+1)^2$	MW1	
	$-\frac{1}{2}2n(2n+1) + \frac{1}{2}n(n+1)$	MW1	
	$= n^2\left(4\left[4n^2 + 4n + 1\right] - \left[n^2 + 2n + 1\right]\right)$		
	$-\left(2n^2 + n\right) + \frac{1}{2}n^2 + \frac{1}{2}n$		
	$= 15n^4 + 14n^3 + \frac{3}{2}n^2 - \frac{1}{2}n$	W1	6

3 **(i)**	$f(x) = (1+x)^n \quad f^{I}(x) = n(1+x)^{n-1}$		**AVAILABLE MARKS**
	$f^{II}(x) = n(n-1)(1+x)^{n-2} \quad f^{III}(x) = n(n-1)(n-2)(1+x)^{n-3}$	M1W1	
	$f(0) = 1 \quad f^{I}(0) = n \quad f^{II}(0) = n(n-1) \quad f^{III}(0) = n(n-1)(n-2)$	M1	
	$f(x) \doteqdot f(0) + xf^{I}(0) = \frac{x^2}{2} f^{II}(0) + \frac{x^3}{6} f^{III}(0) + \ldots$		
	$(1+x)^n \doteqdot 1 + nx + \frac{n(n-1)}{2}x^2 + + n\frac{(n-1)(n-2)}{6}x^3 + \ldots$	M1W1	
(ii)	$\frac{1+x}{(1+2x^2)(1-2x)} = \frac{Ax+B}{1+2x^2} + \frac{C}{1-2x}$	M1W1	
	$1+x = (Ax+B)(1-2x) + C(1+2x^2)$	MW1	
	Put $x = \frac{1}{2} \quad \frac{3}{2} = \frac{3}{2}C \Rightarrow C = 1$ Compare coeff x^2, $0 = -2A + 2C \Rightarrow A = 1$ Compare coeff x, $1 = A - 2B \Rightarrow B = 0$	M1W2	
	$\frac{x}{1+2x^2} + \frac{1}{1-2x}$		
(iii)	$\frac{1+x}{\left(1+2x^2\right)(1-2x)} = \frac{x}{1+2x^2} + \frac{1}{1-2x}$		
	$\left(1+2x^2\right)^{-1} = 1 + (-1)\left(2x^2\right) + \frac{(-1)(-2)}{2}\left(2x^2\right)^2 + \ldots$	M1W1	
	$x\left(1+2x^2\right)^{-1} = x - 2x^3 + \ldots$		
	$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}\left(2x\right)^2 + (-1)\frac{(-2)(-3)}{6}\left(-2x\right)^3 + \ldots$	MW1	
	$= 1 + 2x + 4x^2 + 8x^3 + \ldots$	W1	
	$\therefore$ Result $= 1 + 3x + 4x^2 + 6x^3 + \ldots$	W1	16

				AVAILABLE MARKS
4	**(i)**	Aux.Eq^n $m^2 - 10m + 16 = 0$	MW1	
		$(m-2)(m-8) = 0$		
		$y = Ae^{2x} + Be^{8x}$	M1W1	
		Try $y = ke^{3x}$ for particular integral	M1	
		$9ke^{3x} - 30ke^{3x} + 16ke^{3x} = e^{3x}$	MW1	
		$\Rightarrow k = -\frac{1}{5}$	MW1	
		$y^{gs} = Ae^{2x} + Be^{8x} - \frac{1}{5}e^{3x}$	MW1	
	(ii)	Aux.Eq^n $m^2 + pm + 16$ is a perfect square	MW1	
		$\Rightarrow p = 8$ and $k = -4$ or $p = -8$ and $k = 4$	W3	11
5	**(i)**	Let $P(k) \equiv 2^{k+1}\sin x \cos x \ldots \cos 2^k x \equiv \sin\left(2^{k+1}x\right)$		
		When $n = 0$ LHS = $2\sin x \cos x$		
		RHS = $\sin 2x$		
		$\therefore$ P(0) true ①	M1W1	
		Assume P(k) true	M1	
		Consider $2^{k+2}\sin x \cos x \ldots \cos\left(2^{k+1}x\right)$	M1	
		$= 2\left(2^{k+1}\sin x \cos x \ldots \cos 2^k x\right)\cos 2^{k+1}x$		
		$= 2\sin\left(2^{k+1}x\right)\cos\left(2^{k+1}x\right)$ by P(k)	MW1	
		$= \sin 2^{k+2}x \therefore P(k+1)$ true ②	MW1	
		① and ② $\Rightarrow$ P(n) true $\forall n \in \mathbb{Z}$ $n \geqslant 0$	M1	
	(ii)	$2^3 \sin x \cos x \cos 2x \cos 4x = \frac{\sqrt{2}}{2}$	MW1	
		by (i) $\sin 8x = \frac{1}{\sqrt{2}}$	M1W1	
		$8x = n\pi + (-1)^n \frac{\pi}{4}$	M2W1	
		$x = \frac{n\pi}{8} + (-1)^n \frac{\pi}{32}$	W1	14

				AVAILABLE MARKS
6	(i)	$a = \frac{6}{2} = 3$	MW1	
		With vertex(0, 0) eq^n $y^2 = 12x$	MW1	
		vertex (3,0) $y^2 = 12(x-3)$	MW1	
	(ii)	$y^2 = 36t^2$	M1	
		$12(x-3) = 12(3t^2+3-3) = 36t^2$ equal	W1	
	(iii)	$\frac{dy}{dt} = 6$ $\frac{dx}{dt} = 6t$ $\frac{dy}{dx} = \frac{6}{6t}$ $\frac{dy}{dx} = \frac{1}{t}$	M1W2	
		$y - 6t = \frac{1}{t}(x - 3t^2 - 3)$	M1W1	
		$ty - 6t^2 = x - 3t^2 - 3$		
		$ty - x = 3t^2 - 3$	W1	
	(iv)	Tangent at R $-\frac{y}{t} - x = \frac{3}{t^2} - 3$	M1W1	
		$\Rightarrow -ty - t^2x = 3 - 3t^2$		
		Tangent at T $ty - x = 3t^2 - 3$	M1	
		Add $-\left(1+t^2\right)x = 0$		
		$\therefore x = 0$	W1	15

7 **(i)**



		AVAILABLE MARKS
	MW1	
	MW1	
(ii) $z = 1e^{\frac{1\pi}{4}}$ r = 1	MW1	
$k = \frac{\pi}{4}$	MW1	
(iii) $\left(z^8 - 1\right) \div \left(z^4 - 1\right)$ for $z^4 - 1$	MW1	
$z^8 - 1 = \left(z^4 - 1\right)\left(z^4 + 1\right)$	M1W1	
Equation $z^4 + 1 = 0$	MW1	
Alternative solution		
$\left[z - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right]\left[z - \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)\right]$	M1	
$= z^2 - \sqrt{2}\,z + 1$	W1	
$\left[z - \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right]\left[z - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)\right]$		
$= z^2 + \sqrt{2}\,z + 1$		
$\left(z^2 - \sqrt{2}z + 1\right)\left(z^2 + \sqrt{2}z + 1\right)$	W1	
$= z^4 + \sqrt{2}z^3 + z^2$		
$- \sqrt{2}z^3 - 2z^2 - \sqrt{2}z$		
$+ z^2 + \sqrt{2}z + 1$		
$= z^4 + 1$		
So eq^n is $z^4 + 1 = 0$	MW1	8
	Total	**75**

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Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

THURSDAY 27 MAY, MORNING

MARK SCHEME

Introduction

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Some common examples of this occur in the following cases:

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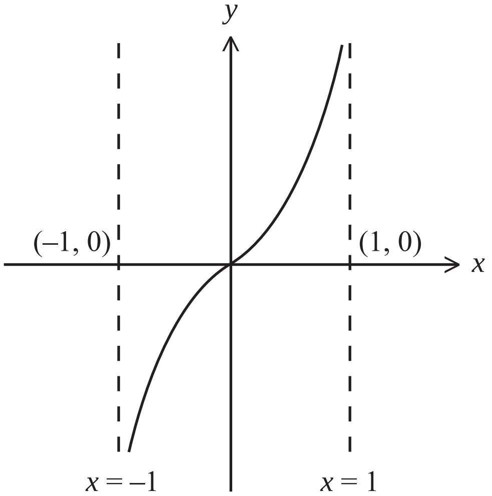
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1	$u = e^x \qquad \dfrac{du}{dx} = e^x$	MW1	AVAILABLE MARKS
	$\dfrac{dv}{dx} = \cos x \qquad v = \sin x$	MW1	
	$\int e^x \cos x\, dx = e^x \sin x - \int e^x \sin x\, dx$	MW1	
	$u = e^x \qquad \dfrac{du}{dx} = e^x$	W1	
	$\dfrac{dv}{dx} = \sin x \qquad v = -\cos x$		
	$\int e^x \cos x\, dx = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x\, dx\right]$		
	$\int e^x \cos x\, dx = e^x \sin x + e^x \cos x - \int e^x \cos x\, dx + c$	MW1	
	$2\int e^x \cos x\, dx = e^x \sin x + e^x \cos x + c$		
	$\int e^x \cos x\, dx = \dfrac{e^x}{2}(\sin x + \cos x) + c'$	MW1	6

				AVAILABLE MARKS
2	**(i)**	Origin in plane – necessary. 2 – 0 – 2 = 0 A in plane 4 – 6 + 2 = 0 B in plane	M1W1	
	(ii)	plane $\prod_2$ $\mathbf{r}\cdot\mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = d$	M1	
		$d = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 12 - 6 - 1 = 5$	W1	
	(iii)	$\prod_2$ Cartesian equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 3x - 2y - z = 5$	M1W1	
		$3x - 2y - z = 5$ (a) $x - 2y + 2z = 0$ (b) $2x - 3z = 5$	M1	
		$x = \dfrac{3z+5}{2}$	W1	
		Substitute in (b) $\dfrac{3z+5}{2} - 2y + 2z = 0$ $4y = 7z + 5$		
		$z = \dfrac{4y-5}{7}$	W1	
		$\therefore \dfrac{2x-5}{3} = \dfrac{4y-5}{7} = z$	W1	10

			AVAILABLE MARKS
3	**(i)** $u = x^n \qquad \frac{du}{dx} = nx^{n-1}$	MW1	
	$\frac{dv}{dx} = e^{-x} \qquad v = -e^{-x}$	MW1	
	$I_n = \int_0^1 x^n e^{-x} dx = \left[-x^n e^{-x}\right]_0^1 + \int_0^1 nx^{n-1} e^{-x} dx$	M1	
	$= -e^{-1} + n\int_0^1 x^{n-1} e^{-x} dx$	W1	
	$\therefore I_n = -e^{-1} + nI_{n-1}$	W1	
	(ii) $I_4 = \int_0^1 x^4 e^{-x} dx = \left[4I_3 - e^{-1}\right]$	M1	
	$= 4\left[3I_2 - e^{-1}\right] - e^{-1} = 12\left(2I_1 - e^{-1}\right) - 5e^{-1}$	W1	
	$= 24\left(I_0 - e^{-1}\right) - 17e^{-1} = 24I_0 - 41e^{-1}$	W1	
	$I_0 = \int_0^1 e^{-x} dx = \left[-e^{-x}\right]_0^1 = \left[-e^{-1} + 1\right]$	MW1	
	$\therefore I_4 = 24 - 65e^{-1}$	W1	10
4	$u = e^x \qquad \frac{du}{dx} = e^x = u \qquad \frac{du}{u} = dx$	MW1	
	$5\cosh x + 4\sinh x = \frac{5\left(e^x + e^{-x}\right)}{2} + \frac{4\left(e^x - e^{-x}\right)}{2}$	M1	
	$= \frac{9e^x + e^{-x}}{2} = \frac{9u + \frac{1}{u}}{2}$		
	$= \frac{9u^2 + 1}{2u}$	W2	
	$\int \frac{dx}{5\cosh + 4\sinh x} = \int \frac{2u}{9u^2 + 1} \cdot \frac{du}{u}$	M1	
	$= \int \frac{2du}{9u^2 + 1} = \int \frac{2}{(3u)^2 + 1} du$	W1	
	$= \frac{2}{3}\tan^{-1}(3u) + c$	M1	
	$= \frac{2}{3}\tan^{-1}\left(3e^x\right) + c$	W1	8

			AVAILABLE MARKS
5	**(i)** $y = \cos^{-1} x \qquad \cos y = x$		
	$-\sin y \dfrac{dy}{dx} = 1$	M1W1	
	$\sin^2 y = 1 - \cos^2 y = 1 - x^2$	MW1	
	$\therefore \dfrac{dy}{dx} = \dfrac{-1}{\sin y} = \dfrac{-1}{\sqrt{1-x^2}}$	W1	
	(ii) $y = \cos^{-1} 4x \qquad \dfrac{dy}{dx} = \dfrac{-4}{\sqrt{1-16x^2}}$	M1W1	
	$x = \dfrac{1}{8} \quad y = \cos^{-1}\dfrac{1}{2} = \dfrac{\pi}{3}$	MW1	
	$x = \dfrac{1}{8} \quad \dfrac{dy}{dx} = \dfrac{-4}{\sqrt{1 - 16 \cdot \dfrac{1}{64}}} = \dfrac{-8}{\sqrt{3}} = \dfrac{-8\sqrt{3}}{3}$	MW1	
	$y - \dfrac{\pi}{3} = -\dfrac{8\sqrt{3}}{3}\left(x - \dfrac{1}{8}\right)$	MW1	
	$x = 0 \quad y = OP = \dfrac{\pi}{3} + \dfrac{\sqrt{3}}{3} = \dfrac{\pi + \sqrt{3}}{3}$	MW1	10
6	**(i)** $y = \tanh^{-1} x \qquad \tanh y = x = \dfrac{\sinh y}{\cosh y}$	MW1	
	$\therefore x = \dfrac{e^y - e^{-y}}{e^y + e^{-y}} = \dfrac{e^{2y} - 1}{e^{2y} + 1}$	M1W1	
	$x\left(e^{2y} + 1\right) = e^{2y} - 1$		
	$1 + x = e^{2y}\left(1 - x\right)$	W1	
	$e^{2y} = \dfrac{1+x}{1-x}; 2y = \ln\left(\dfrac{1+x}{1-x}\right)$		
	$y = \tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	W1	
	(ii) 	W1 W1	

		AVAILABLE MARKS
(iii) $y = \tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \frac{1}{2}\left[\ln(1+x) - \ln(1-x)\right]$	M1W1	
$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+x} - \frac{-1}{1-x}\right]$	MW1	
$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{2}{1-x^2} = \frac{1}{1-x^2}$	W1	
or		
$\tanh y = x$	M1	
$\operatorname{sech}^2 y \frac{dy}{dx} = 1$	W1	
$\operatorname{sech}^2 y \equiv 1 - \tanh^2 y = 1 - x^2$	MW1	
$(1-x^2)\frac{dy}{dx} = 1$		
$\frac{dy}{dx} = \frac{1}{1-x^2}$	W1	
(iv) $x = \tanh\left[\ln\left(\sqrt{6x}\right)\right]$		
$\tanh^{-1} x = \ln\left(\sqrt{6x}\right)$	M1	
$\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \frac{1}{2}\ln(6x)$	W1	
$\frac{1+x}{1-x} = 6x$	MW1	
$6x^2 - 5x + 1 = 0$	W1	
$(3x-1)(2x-1) = 0$		
$x = \frac{1}{3}$ or $x = \frac{1}{2}$	W1	16
7 **(i)** Area of triangle $= \frac{1}{2}\lvert \mathbf{a}\times\mathbf{b} \rvert$	M1	
$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = -5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$	M1W2	
$\lvert \mathbf{a}\times\mathbf{b} \rvert = \sqrt{35}$ Area of triangle $= \frac{\sqrt{35}}{2}$	MW1	

		AVAILABLE MARKS
(ii) **line** $(\mathbf{r}-\mathbf{a})\times\mathbf{m}=\mathbf{0}$ has equation $\mathbf{r}=-\mathbf{i}+\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}+\mathbf{k})$	MW1	
line $(\mathbf{r}-\mathbf{b})\times\mathbf{n}=\mathbf{0}$ has equation $\mathbf{r}=-\mathbf{i}+2\mathbf{j}-\mathbf{k}+\mu(-\mathbf{i}-\mathbf{j}+4\mathbf{k})$	MW1	
Intersect hen $-\mathbf{i}+\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}+\mathbf{k})=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\mu(-\mathbf{i}-\mathbf{j}+4\mathbf{k})$	M1	
Equate $\left.\begin{array}{r}-1+\lambda=1-\mu \\ 1=2-\mu\end{array}\right\}-\mu=1 \quad \lambda=1$	MW2	
$2+\lambda=1+4\mu$		
LHS $=2+\lambda=3$ RHS $=-1+4\mu=3$	MW1	
Position vector of C is $\mathrm{c}=\mathbf{j}+3\mathbf{k}$		
Accept C is the point (0, 1, 3)	W1	
(iii) Volume $=\frac{1}{6}\left\lvert \mathrm{c}\cdot(\mathrm{a}\times\mathrm{b})\right\rvert$		
$=\frac{1}{6}\left\lvert(\mathbf{j}+3\mathbf{k})\cdot(-5\mathbf{i}+\mathbf{j}-3\mathbf{k})\right\rvert$	M1W1	
$=\frac{1}{6}\lvert 1-9\rvert=\frac{8}{6}=\frac{4}{3}$	W1	15
	Total	**75**

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Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

FRIDAY 11 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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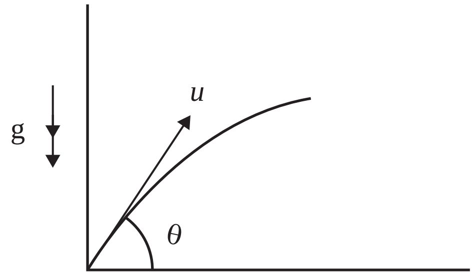
			Marks	AVAILABLE MARKS
1	**(i)**	$\mathbf{F_1} + \mathbf{F_2} = (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ N	MW1	
		$\mathbf{F} = m\mathbf{a}$	M1	
		$3\mathbf{i} + \mathbf{j} - \mathbf{k} = 2\mathbf{a}$		
		$\mathbf{a} = \frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$	W1	
	(ii)	Resultant Force = $3\mathbf{i} + \mathbf{j} - \mathbf{k}$		
		$\mathbf{F}.\mathbf{F_1} = \lvert\mathbf{F}\rvert\lvert\mathbf{F_1}\rvert\cos\theta$	M1	
		$(3\mathbf{i} + \mathbf{j} - \mathbf{k}).(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 6 - 2 - 1 = 3$	MW1	
		$\lvert\mathbf{F}\rvert = \sqrt{9+1+1} = \sqrt{11}$	M1W1	
		$\lvert\mathbf{F_1}\rvert = \sqrt{4+4+1} = 3$	MW1	
		$\cos\theta = \frac{3}{3\sqrt{11}} = \frac{1}{\sqrt{11}}$		
		$\theta = 72.5°$	W1	9
2	**(i)**	$\text{KE} = \frac{1}{2}mv^2$	M1	
		At start KE $\frac{1}{2} \times 0.05 \times 100 = 2.5$ J	W1	
		At end KE $\frac{1}{2} \times 0.05 \times 16 = 0.4$ J	W1	
		Change in KE = 2.5J – 0.4J Decrease of 2.1 J	W1	
	(ii)	Work done by gravity = mgd	M1	
		$= 0.05 \times 9.8 \times 2$		
		$= 0.98$ J	W1	
	(iii)	By work – energy principle		
		Work done by resultant force = Change in KE	M1	
		Resultant force = $mg - R$	MW1	
		$(mg - R)d = -2.1$		
		$[(0.05 \times 9.8) - R](2) = -2.1$	W2	
		$R = 1.54$ N	W1	11

3 **(i)**



AVAILABLE MARKS

At greatest height $v = 0$ M1

$u = u \sin \theta$

$v = 0$ $v^2 = u^2 + 2as$ M1

$a = -g$ $0 = u^2 \sin^2 \theta - 2gs$

$s = ?$ $s = \dfrac{u^2 \sin^2 \theta}{2g}$ W1

(ii) When travelling horizontally $v = 0$ vertically M1

$u = u \sin \theta$

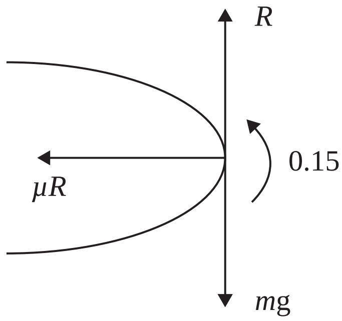
$v = 0$ $v = u + at$ M1

$a = -g$ $0 = u \sin \theta - gt$

$t = ?$ $t = \dfrac{u \sin \theta}{g}$ W1 6

4 **(i)**

r = 100 m

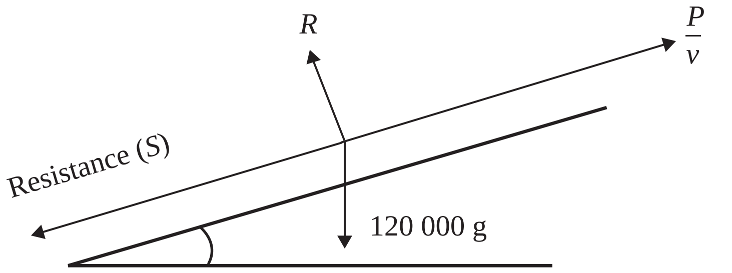


$\mu R = mr\omega^2$	M3	
$\mu mg = mr\omega^2$	MW1	
$\mu = \dfrac{100(0.15)^2}{10} = 0.225$	W1	
(ii) Kinetic Energy $= \dfrac{1}{2}mv^2$	M1	
$v = r\omega$	M1	
$KE = \dfrac{1}{2}m(100)^2(0.15)^2$	W1	
$= 113m$ J	W1	9

AVAILABLE MARKS

		AVAILABLE MARKS

5 **(i)**



MW2

(ii) At steady speed $a = 0$ — M1

$P = Fv$

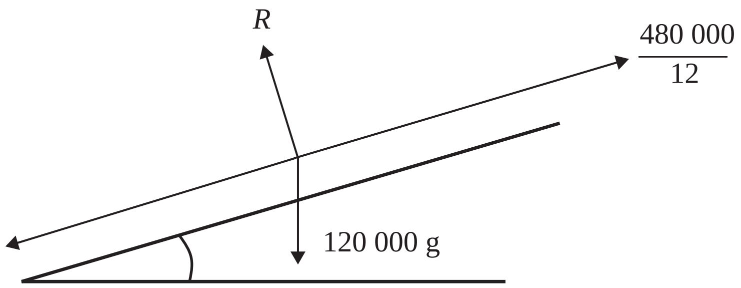
$F = \frac{P}{v} = \frac{240\,000}{12} = 20\,000\text{ N}$ — M1W1

Using $F = ma$ — M1

$20\,000 \quad S \mp 120\,000\text{ g} \sin \theta$ — W1

$S = 10\,200\text{ N}$ — W1

(iii)

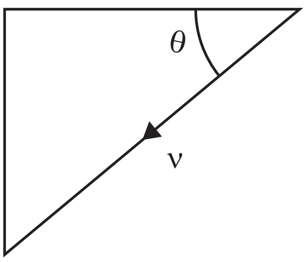


Using $F = ma$

$\frac{480\,000}{12} - 10\,200 - \frac{120\,000\,(9.8)}{120} = 120\,000\,a$ — MW3

$a = 0.167\text{ m s}^{-2}$ — W1 — 12

				AVAILABLE MARKS
6	**(i)**	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (3t^2 - 6t)\mathbf{i} + (2t - 4)\mathbf{j}$	M1W2	
	(ii)	For $\mathbf{a} = 0$ $\quad$ **i** and **j** components equal zero	M1	
		$(3t^2 - 6t) = 0 \quad t = 0$ or $t = 2$		
		$(2t - 4) = 0 \quad t = 2$	MW1	
		$t = 2$ seconds	W1	
	(iii)	$\mathbf{s} = \int \mathbf{v}\,dt$	M1	
		$\mathbf{s} = \left(\frac{t^4}{4} - t^3\right)\mathbf{i} + \left(\frac{t^3}{3} - 2t^2\right)\mathbf{j} + \mathbf{c}$	W2	
		At $t = 0$, $\mathbf{s} = 3\mathbf{j} \quad \therefore \mathbf{c} = 3\mathbf{j}$		
		$\mathbf{s} = \left(\frac{t^4}{4} - t^3\right) + \left(\frac{t^3}{3} - 2t^2 + 3\right)\mathbf{j}$	MW1	
	(iv)	When $t = 2$		
		$\mathbf{s} = -4\mathbf{i} - \frac{7}{3}\mathbf{j}$	MW1	
		$\lvert\mathbf{s}\rvert = \sqrt{16 + \frac{49}{9}}$	M1	
		$= 4.63$ m	W1	
	(v)	When $t = 2$		
		$\mathbf{v} = -4\mathbf{i} - 4\mathbf{j}$	MW1	
		$\theta = \tan^{-1} 1$	M1W1	
		At 135° below positive **i** direction	MW1	17



				AVAILABLE MARKS
7	**(i)**	$a = \dfrac{1}{(s-600)^2}$		
		$v\dfrac{dv}{ds} = \dfrac{1}{(s-600)^2}$	MW1	
		$\int v\ dv = \int \dfrac{1}{(s-600)^2}\ ds$	M2W1	
		$\dfrac{v^2}{2} = \dfrac{-1}{s-600} + c$	MW2	
		At $v = 0,\ s = 0,\ c = -\dfrac{1}{600}$	M1W1	
		$\dfrac{v^2}{2} = \dfrac{1}{600-s} - \dfrac{1}{600}$		
		$v = \sqrt{\dfrac{2}{600-s} - \dfrac{1}{300}}$		
		$v = \sqrt{\dfrac{s}{300(600-s)}}$	MW1	
	(ii)	By considering the motion at B, velocity and acceleration at B would be infinite using this model.	MW2	11
			Total	**75**

Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

TUESDAY 15 JUNE, MORNING

MARK SCHEME

Introduction

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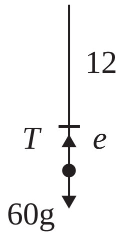
				AVAILABLE MARKS
1	**(i)**	AB = 30, AD = 24		
		By Pythagoras BD = 18	M1W1	
		$\triangle$s similar, scale $\frac{1}{2}$ $\therefore$ BH = 9	MW1	
	(ii)	Let m be the mass of $\triangle$ADE		
			M1	
		$4m \times 3 - m \times -3 = 3md$	M1W2	
		$d = 5$	W1	8
2	**(i)**	$a = 1$, $a\omega^2 = 9$	MW2	
		and $\omega > 0$ $\therefore$ $\omega = 3$	W1	
		$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$	M1W1	
	(ii)	$v^2 = \omega^2(a^2 - x^2)$	M1	
		$= 9(1 - 0.64) = 9 \times 0.36$	W1	
		$v > 0$ $\therefore$ $v = 3 \times 0.6 = 1.8\ \mathrm{m\,s^{-1}}$	W1	
	(iii)	Starting at A, $t = 0$	M1	
		$x = a \cos \omega t$	MW1	
		$-0.8 = 1 \cos 3t$	MW1	
		$3t = 2.498$	M1	
		$t = 0.833\ \mathrm{s}$	W1	13

AVAILABLE MARKS

3 **(i)**



Res $\updownarrow$ $T = mg = 60g$ — MW1

By Hooke's Law $T = \frac{240g}{12}$ — M1W1

$20g\, e = 60g$

$e = 3$ — W1

(ii) equating energies at 2 pts — M1

at clifftop GPE = $60g \times 15$ EPE = 0   KE = 0 — W1

at P   GPE = $60g\, d$, EPE = $\frac{1}{2}\frac{240g}{12}(3 + d)^2$ KE = 0 — M1W2

$10g(9 + 6d + d^2) - 60g\, d = 900g$

$90 + 10d^2 = 900$

$10d^2 = 810$

$d^2 = 81$ and $d > 0$ $\therefore$ $d = 9$ — W1

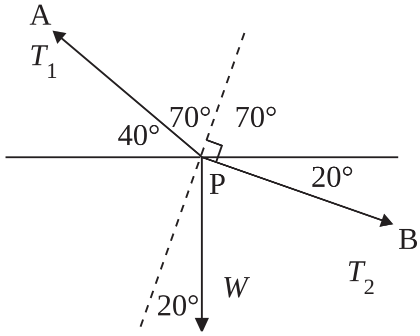
(iii) Res $\updownarrow$ $R = T - 60g$ — M1

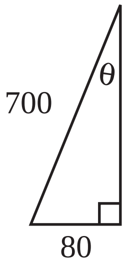
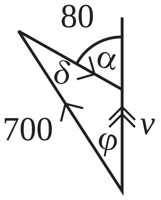
at P, extn is 12 m $\therefore$ $T = \frac{240g}{12} \times 12 = 240g$ — MW1

$R = 240g - 60g$

$= 180g$ N — W1 — 13

		AVAILABLE MARKS
4 (a) (i) $W = \int_0^d 3(x-2)^2 \, dx$	M1W1	
$= \left[(x-2)^3\right]_0^d$	MW1	
$= (d-2)^3 - (-8)$		
$= (d-2)^3 + 8$	W1	
(ii) $(d-2)^3 + 8 = 35$	MW1	
$(d-2)^3 = 27$		
$d - 2 = 3$		
$d = 5$	W1	
(b) (i) $D = \mathbf{F}.\mathbf{s}$	M1	
$= 3\begin{pmatrix} 2 \\ -5 \\ a \end{pmatrix} . \begin{pmatrix} a-1 \\ 1-2a \\ a \end{pmatrix}$	M1	
$= 3(2a - 2 - 5 + 10a + a^2)$		
$= 3(a^2 + 12a - 7)$	W1	
(ii) $3(a^2 + 12a - 7) = 114$	MW1	
$a^2 + 12a - 7 = 38$		
$a^2 + 12a - 45 = 0$		
$(a-3)(a+15) = 0$		
$a = 3$ or -15	W2	
(iii) $a = 3$		
$\mathbf{F} = 3\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}, \ \mathbf{s} = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$	MW2	
(iv) For $a = 3$, **F** and **s** are in the same direction so the resultant **R** of the system of forces is in the direction of **F**.	MW2	16

			AVAILABLE MARKS
5	**(i)** $T = \frac{\lambda x}{l} \quad \therefore \; x = \frac{Tl}{\lambda}$	M1MW1	
	$= \frac{0.8 \times 92}{184}$		
	$= 0.4\,\text{m}$	W1	
	(ii) 		
	$T_1 \cos 70° = W \cos 20°$	M1W2	
	$W = \frac{92 \cos 70°}{\cos 20°}$		
	$= 33.485$		
	$\rightarrow 33.5\,\text{N}$	W1	
	(iii) Res $\leftrightarrow$	M1	
	$T_1 \cos 40° = T_2 \cos 20°$	MW1	
	$T_2 = \frac{92 \cos 40°}{\cos 20°}$		
	$= 74.999$		
	$\rightarrow 75.0\,\text{N}$	W1	
	(iv) $x = \frac{1}{4}(0.9 + x)$	MW1	
	$\frac{3}{4}x = \frac{1}{4} \times 0.9$		
	$x = 0.3$	MW1	
	$75 = \frac{\lambda \times 0.3}{0.9}$		
	$\lambda = 225\,\text{N}$	MW1	13

		AVAILABLE MARKS
6 **(i)**  $\sin\theta° = \frac{8}{70} \quad \therefore\ \theta = 6.56°$	M1W1	
(ii) ${}_{P}\mathbf{V}_{W} + {}_{W}\mathbf{V}_{E} = {}_{P}\mathbf{V}_{E}$	M1	
	MW1	
$\frac{\sin\varphi}{80} = \frac{\sin(180 - \alpha)°}{700}$	M1	
$= \frac{\sin\alpha}{700} = \frac{0.8}{700}$	W1	
$\sin\varphi = \frac{64}{700}$		
$\varphi = 5.246° \rightarrow 5.25°$	W1	
Bearing is $360° - \varphi = 355°$	MW1	
(iii) $\alpha = \delta + \varphi \quad \therefore\ \delta = \alpha - \varphi = 47.8843°$	MW1	
$\frac{v}{\sin\delta} = \frac{700}{\sin\alpha}$	MW1	
$v = \frac{700\sin\delta}{\sin\alpha} = 649.068\ \text{kh}^{-1}$	W1	
$t = \frac{2000}{v}$		
$= 3.08$ hours	MW1	12
	Total	**75**

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Mathematics

Assessment Unit M4
assessing
Module M4: Mechanics 4

[AMM41]

FRIDAY 18 JUNE, AFTERNOON

MARK SCHEME

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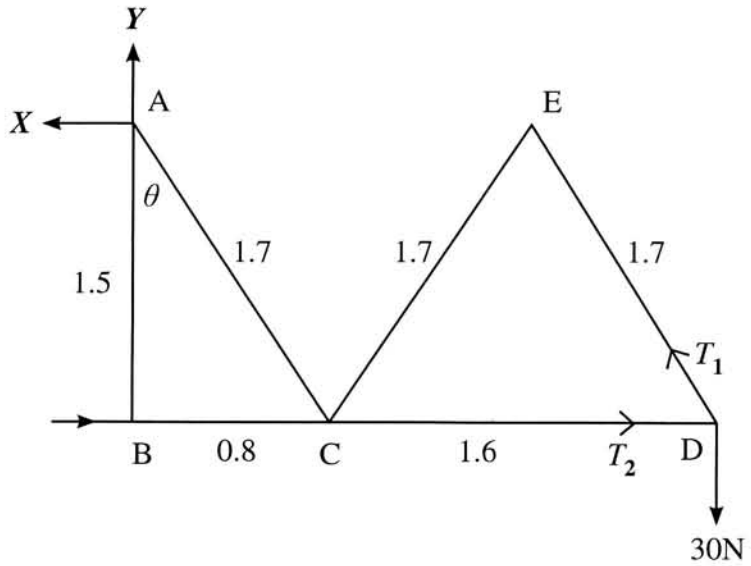
1 **(i)**	mass = $\rho \times$ area so $m = \rho\int_0^4 y\,\mathrm{d}x = \rho\int_0^4 (4-x)^{\frac{1}{2}}\,\mathrm{d}x$	M2W1	
	$= \frac{2}{3}\rho\left[(-1)(4-x)^{\frac{3}{2}}\right]_0^4$	W1	
	$= \frac{2}{3}\rho \times 8$		
	$= \frac{16\rho}{3}$	W1	
(ii)	c.o.m. of strip is $\left(x, \frac{y}{2}\right)$	MW1	
	$\therefore\ \mathcal{M}_{0x} = \rho\int_0^4 \frac{y}{2} \times y\,\mathrm{d}x = \frac{\rho}{2}\int_0^4 y^2\mathrm{d}x$	MW1	
	$= \frac{\rho}{2}\int_0^4 (4-x)\,\mathrm{d}x$	W1	
(iii)	$\mathcal{M}_{0x} = \frac{\rho}{2}\left[\frac{-(4-x)^2}{2}\right]_0^4$	MW1	
	$= \frac{\rho}{4}\left[0-(-16)\right]$	W1	
	$= 4\rho$		
	$\overline{y} = \frac{\mathcal{M}_{0x}}{M} = \frac{4\rho \times 3}{16\rho} = \frac{3}{4}$	MW1	11

AVAILABLE MARKS

2



(i) ED must have an upwards vertical component to balance the load. So a tension T_1 So ED has an inwards horizontal component and so the force in CD must act out, so a thrust T_2 — M1, M1

(ii) $\cos\theta = \frac{15}{17}$ — MW1

$T_1 \cos\theta = W \quad \frac{15}{17}T_1 = 30 \quad T_1 = 34\,\text{N}$ — M1W1

$T_1 \sin\theta = T_2 \quad T_2 = 34 \times \frac{8}{17} = 16\,\text{N}$ — MW1

(iii) The only action at B is horizontal from the rod CB and B is in equilibrium. Only a horizontal force can balance this. — M1

(iv) $X \leftarrow \uparrow Y$ Consider the system

Res $Y = 30\,\text{N}$ — MW1

$\mathcal{M}_B \quad 1.5X = 2.4 \times 30$ — M1W1

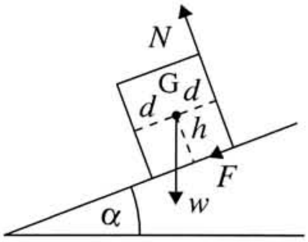
$X = 48\,\text{N}$ — W1

$R_A = \sqrt{30^2 + 48^2} = 56.6\,\text{N}$ — MW1 — 12

AVAILABLE MARKS

3 **(i)** The wheels have just lifted off the road so *normal reaction* = 0 — M1

(ii)



$\mathcal{M}_G$ $Nd = Fh$ — M2W1

$F = \mu N$ — MW1

$\mu = \dfrac{d}{h}$ — MW1

(iii) $\mu = \dfrac{d}{1.25d} = 0.8$ — MW1

(iv) $v^2 = rg\left(\dfrac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) = 0.990066 \times 196$ — M1W1

$v = 13.93 \rightarrow 13.9 \text{ m s}^{-1}$ — W1 — 10

4 **(i)** $\underset{\rightarrow}{20}$ $\underset{\rightarrow}{0}$

(1) (m)

$\underset{-v}{\rightarrow}$ $\underset{2v}{\rightarrow}$

Conservation of Momentum $2mv - v = 20$ — M1W1

$2mv = 20 + v$

$m = \dfrac{20 + v}{2v}$ — W1

(ii) P's loss $\dfrac{1}{2} \times 1 \times 20^2 - \dfrac{1}{2} \times 1 \times v^2$ — M1W1

$\therefore \dfrac{1}{4}\left(20^2 - v^2\right) = \dfrac{1}{2}m \times 4v^2 = 2mv^2$ — M1W1

$= v\left(20 + v\right)$

$400 - v^2 = 80v + 4v^2$

$5v^2 + 80v - 400 = 0$ — W1

$v^2 + 16v - 80 = 0$

$(v - 4)(v + 20) = 0$

$v = 4 \text{ or} -20, \text{ but } v > 0$

$\therefore v = 4$ — W1

(iii) Newton's Law of Restitution — M1

$2v - (-v) = -e(0 - 20)$ — W1

$3v = 20e$

$20e = 12$

$e = 0.6$ — MW1 — 12

AVAILABLE MARKS

5 **(i)** Take G.P.E. = 0 at B

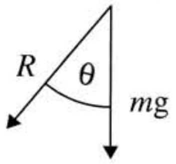
$\therefore$ at A Total Energy $= \frac{1}{2}m \times 2gr + mg \times 2r = 3mgr$ — M1W1

$\therefore$ at P Total Energy $= \frac{1}{2}mv^2 + mgr(1+\cos\theta)$ — MW2

$\therefore$ equating energies $\frac{1}{2}mv^2 = 2mgr - mgr\cos\theta$ — M1

$mv^2 = 2mgr(2-\cos\theta)$ — W1

Res $\swarrow$ $\frac{mv^2}{r} = R + mg\cos\theta$ — M2W1



$R = \frac{mv^2}{r} - mg\cos\theta$

$= 4mg - 3mg\cos\theta$

$= mg(4-3\cos\theta)$ — W1

(ii) $-1 \leqslant \cos\theta \leqslant 1 \therefore 1 \leqslant (4-3\cos\theta) \leqslant 7$ — MW1

i.e. $R > 0$ so R always points to C — MW1

(iii) $R_{min} = 1mg$, $R_{max} = 7mg$ — MW2 — 14

$\therefore R_{max} = 7R_{min}$

Alternative solution

(i) By W.E.P. Work done = change in K.E. — M1

$W = (2r - r - r\cos\theta)mg$ — MW1

$= rmg(1-\cos\theta)$ — W1

Change in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ — M1

$= \frac{1}{2}mv^2 - mgr$ — W1

$\frac{1}{2}mv^2 - mgr = mgr - mgr\cos\theta$

$\frac{1}{2}mv^2 = 2mgr - mgr\cos\theta$

$mv^2 = 4mgr - 2mgr\cos\theta$ — W1

Res $\swarrow$ $R + mg\cos\theta = \frac{mv^2}{r}$ — M1M1W1

$R = 4mg - 3mg\cos\theta$

$= mg(4-3\cos\theta)$ — W1

		AVAILABLE MARKS
6 **(i)** $P = kn^s I^t l^u r^v$		
$[\mathrm{T}] = [\mathrm{ML^{-1}T^{-2}}]^s[\mathrm{ML^2}]^t[\mathrm{L}]^u[\mathrm{L}]^v$	M1W1	
equating indices [T], $1 = -2s \ \therefore s = -\dfrac{1}{2}$	M1W1	
equating indices [M], $0 = s + t$	MW1	
$t = -s = \dfrac{1}{2}$	W1	
(ii) equating indices [L], $0 = -s + 2t + u + v$	MW1	
$0 = \dfrac{1}{2} + 1 + u + v$		
$v = -u - \dfrac{3}{2}$	MW1	
(iii) $P = kn^{-\frac{1}{2}} I^{\frac{1}{2}} l^u r^{-\frac{3}{2}-u}$	M1	
so $P = k\sqrt{\dfrac{I}{nr^3}}\left(\dfrac{l}{r}\right)^u$	W1	
(iv) $1.7725 = k\sqrt{\dfrac{I}{nr^3}}\left(\dfrac{0.4^u}{r^u}\right)$	MW1	
$3.545 = k\sqrt{\dfrac{I}{nr^3}}\left(\dfrac{1.6^u}{r^u}\right)$	MW1	
$2 = \dfrac{1.6^u}{0.4^u} = 4^u$	MW1	
$\therefore u = \dfrac{1}{2}$	MW1	
(v) $1.7725 = k\sqrt{\dfrac{2.2\times10^{-5}\times0.4}{4.4\times10^{10}\times16\times10^{-16}}}$	MW1	
$k = 1.7725\times\sqrt{8}$		
$= 5.013$		
$\to 5.01$	W1	16
	Total	**75**

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Mathematics

Assessment Unit S4

assessing

Module S2: Statistics 2

[AMS41]

FRIDAY 18 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

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AVAILABLE MARKS

1 **(i)** $r = \dfrac{\sum xy - \dfrac{(\sum x)(\sum y)}{n}}{\sqrt{\sum x^2 - \dfrac{(\sum x)^2}{n}}\ \sqrt{\sum y^2 - \dfrac{(\sum y)^2}{n}}}$ M1

$= \dfrac{233000 - \dfrac{1364 \times 1361}{8}}{\sqrt{233936 - \dfrac{1364^2}{8}}\ \sqrt{232343 - \dfrac{1361^2}{8}}}$ MW1 MW2

$= 0.90401916\ldots = 0.904$ (3sf) W1

(ii) Strong positive correlation between the heights of twenty-year-old sons and the heights of their father. M1 — 6

2 **(i)** The response variable is to be measured M1
against set values and is subject to variation. M1
In Hooke's Law experiment the extension is the variable measured in response to the mass. M1

(ii) Other factors remain unchanged e.g. all M1
readings at the same temperature M1

(iii) $S_{xx} = \sum x^2 - \dfrac{(\sum x)^2}{n}$

$= 34750000 - \dfrac{(13500)^2}{6} = 4375000$ MW1

$S_{xy} = \sum xy - \dfrac{(\sum x)(\sum y)}{n}$

$= 45680 - \dfrac{13500 \times 17.73}{6} = 5787.5$ MW1

$b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{5787.5}{4375000} = 1.32 \times 10^{-3}$ (3sf) MW1

$a = \bar{y} - b\bar{x} = \dfrac{17.73}{6} - (1.32 \times 10^{-3}) \times \dfrac{13500}{6}$ MW1

$= -0.021428571\ldots = -0.0214$ (3sf) W1

$y = (1.32 \times 10^{-3})\,x - 0.0214$ W1

			AVAILABLE MARKS
	(iv) If x = 2200, then		
	$y = 1.32 \times 10^{-3} \times 2200 - 0.0214$	M1	
	y = 2.88% (3sf)	W1	13
3	**(i)** $\bar{x} = \frac{\sum x}{n} = \frac{1721}{60} = 28.7$ (3sf)	MW1	
	$\hat{\sigma}^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$	M1	
	$= \frac{1}{59}\left(49441 - \frac{1721^2}{60}\right)$		
	= 1.30 (3sf)	W1	
	(ii) $H_0 : \mu = 30$	M1	
	$H_1 : \mu \neq 30$	M1	
	2 tailed test at 5%	M1	
	reject H_0 if $\lvert Z_{test} \rvert > 1.96$	MW1	
	$z_{test} = \frac{\bar{x} - \mu}{\sqrt{\frac{\hat{\sigma}^2}{n}}}$	M1	
	$= \frac{28.7 - 30}{\sqrt{\frac{1.30}{60}}}$	MW1 MW1	
	= − 8.83 (–8.9 if use exact value)	W1	
	Since $\lvert z_{test} \rvert > 1.96$ we reject H_0 and conclude that there is sufficient evidence at 5% level to suggest that the train does not take the 30 minutes that it should.	M1 M1	13

				AVAILABLE MARKS
4	**(i)**	There is a probability of 0.95 (95%) that the interval contains the true value of the population mean.	M2	
	(ii)	The parent distribution is normally distributed	M1	
	(iii)	$\bar{x} = \frac{\sum x}{n} = \frac{339}{45} = 7.53$ (3sf)	MW1	
		$\sigma^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$	M1	
		$= \frac{1}{44}\left(2799 - \frac{339^2}{45}\right) = 5.57$ (3sf)	W1	
	(iv)	$\text{CI} = \bar{x} \pm 1.96\sqrt{\frac{\sigma^2}{n}}$	M1	
		$\text{CI} = \left(\frac{339}{45} - 1.96\sqrt{\frac{5.57}{45}},\ \frac{339}{45} + 1.96\sqrt{\frac{5.57}{45}}\right)$	MW2	
		$= (6.84376\ldots, 8.2230\ldots)$		
		CI = (6.84, 8.22) minutes	W2	11

AVAILABLE MARKS

5

Student	A	B	C	D	E	F	G	H	I	J
Initial Score	56	67	48	70	38	66	54	70	45	51
Final Score	63	75	61	74	51	75	67	73	53	62
d	7	8	1	34	1	39	1	33	8	11

M1W1

From calculator

$\bar{d} = 8.9 \qquad \sigma_{n-1} = 3.63$ (3sf) MW2

$H_0 : \mu_d = 10$ M1

$H_1 : \mu_d < 10$ M1

One tailed test at 5% M1

Using t – test at 9 degrees of freedom M1

Reject H_0 if $t_{\text{test}} < -1.833$ MW1

$$t_{\text{test}} = \frac{\bar{d} - \mu_d}{\hat{\sigma}_d / \sqrt{n}}$$ M1

$$= \frac{8.9 - 10}{3.63 / \sqrt{10}}$$ MW2

$= -0.958$ (3sf) W1

Since $t_{\text{test}} > -1.833$ we do not reject H_0 and conclude that there is insufficient evidence at 5% level to suggest that the program maker's claim is incorrect. M1 M1

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			AVAILABLE MARKS
6	$\overline{X}_{60} \sim \mathrm{N}\left(35, \frac{12}{60}\right) = \mathrm{N}(35, 0.2)$	MW2	
	$\mathrm{P}(\overline{X}_{60} < 34.7) = \mathrm{P}(Z < \frac{34.7 - 35}{\sqrt{0.2}}) = \mathrm{P}(Z < -0.671)$	MW1	
	$= 1 - \Phi(0.671)$	M1	
	$= 1 - 0.7489$	W1	
	$= 0.2511$		
	$= 0.251$ (3sf)	W1	6
7	**(i)** $X + Y \sim \mathrm{N}(20 + 25, 6 + 4) = \mathrm{N}(45, 10)$	M1W1	
	$\mathrm{P}(X + Y > 43) = \mathrm{P}(Z > \frac{43 - 45}{\sqrt{10}}) = \mathrm{P}(Z > -0.632)$	MW1	
	$= \Phi(0.632)$		
	$= 0.7363$		
	$= 0.736$ (3sf)	W1	
	(ii) $\mathrm{P}(3X < 2Y) = \mathrm{P}(3X - 2Y < 0)$	M1	
	$3X - 2Y \sim \mathrm{N}(3 \times 20 - 2 \times 25, 9 \times 6 + 4 \times 4)$	MW1	
	$= \mathrm{N}(10, 70)$	MW2	
	$\mathrm{P}(3X - 2Y < 0) = \mathrm{P}(Z < \frac{0 - 10}{\sqrt{70}}) = \mathrm{P}(Z < -1.195)$	MW1	
	$= 1 - \Phi(1.195)$	M1	
	$= 1 - 0.8840$		
	= 160.1		
	$= 0.116$ (3sf)	W1	11
		Total	**75**