Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

AMF31

THURSDAY 27 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Using integration by parts, find

$$\int e^x \cos x \, dx$$ [6]

2 The points A (2, 0, –1), B (4, 3, 1) and the origin determine a plane Π_1

(i) **Verify** that an equation of the plane is

$$x - 2y + 2z = 0$$ [2]

The plane Π_2 has an equation $\mathbf{r} \cdot \mathbf{n} = d$ where

$$\mathbf{n} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

and d is a constant.

(ii) If the point B lies on the plane Π_2, find the value of d. [2]

The planes Π_1 and Π_2 intersect in the line L.

(iii) Find, in Cartesian form, an equation for the line L. [6]

3 For each non-negative integer n, let

$$I_n = \int_0^1 x^n \mathrm{e}^{-x} \mathrm{d}x$$

(i) Show that for $n \geqslant 1$

$$I_n = nI_{n-1} - \mathrm{e}^{-1}$$ [5]

(ii) Hence find the exact value of

$$\int_0^1 x^4 \mathrm{e}^{-x} \mathrm{d}x$$ [5]

4 By using the substitution $u = \mathrm{e}^x$ find

$$\int \frac{\mathrm{d}x}{5\cosh x + 4\sinh x}$$ [8]

5 **(i)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$ [4]

The tangent at the point where $x = \frac{1}{8}$ on the curve $y = \cos^{-1} 4x$, cuts the y-axis at the point P.

(ii) Show that

$$\mathrm{OP} = \frac{\pi + \sqrt{3}}{3}$$

where O is the origin. [6]

6 **(i)** Using the exponential definitions of sinh x and cosh x show that for $|x| < 1$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$ [5]

(ii) Sketch the graph of $y = \tanh^{-1} x$ clearly labelling any asymptotes. [2]

(iii) Find $\frac{\mathrm{d}}{\mathrm{d}x} (\tanh^{-1} x)$ [4]

(iv) Solve the equation

$$x = \tanh \left(\ln \sqrt{6x} \right)$$

where $0 < x < 1$ [5]

7 Relative to an origin O the points A and B have position vectors **a** and **b** respectively where

$$\mathbf{a} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

and $$\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

(i) Find the area of the triangle OAB. [5]

The point C is the intersection of the two lines

$$(\mathbf{r} - \mathbf{a}) \times \mathbf{m} = \mathbf{0}$$

and $$(\mathbf{r} - \mathbf{b}) \times \mathbf{n} = \mathbf{0}$$

where $\mathbf{m} = \mathbf{i} + \mathbf{k}$ and $\mathbf{n} = -\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

(ii) Find **c**, the position vector of the point C. [7]

(iii) Show that the volume of the tetrahedron OABC is $\frac{4}{3}$ [3]

[The volume of a tetrahedron is $\frac{1}{6} |\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|$]