Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

AMC41

MONDAY 24 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Points P, Q and R have position vectors

$$\overrightarrow{OP} = 4\mathbf{i} + 4\mathbf{j}$$
$$\overrightarrow{OQ} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$
$$\overrightarrow{OR} = 8\mathbf{j} + 6\mathbf{k}$$

(i) Find $\overrightarrow{QP}$. [2]

(ii) Find $\overrightarrow{QR}$. [1]

(iii) Show that the triangle PQR is right-angled at Q. [3]

2 A vase is formed when the area bounded by the curve

$$y = 3 + 2\sqrt{x}$$

and the lines $x = 0$ and $x = 4$ is rotated through 360° about the x-axis, as shown in **Fig. 1** below.

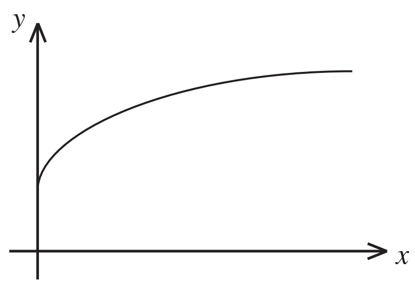

Fig. 1

Find the volume of the vase. [7]

3 **(a)** The function $f(x) = x^2 + 4x$ with domain $\{x: x \in \mathrm{R}, x \geqslant a\}$ is a one-to-one function. By sketching this function, find the least value of a. [3]

(b) The function $g(x) = \dfrac{4x}{x-3}$ has domain $\{x: x \in \mathrm{R}, x \neq b\}$.

(i) Write down the value of b. [1]

(ii) Find the inverse function $g^{-1}(x)$ stating its domain. [5]

(iii) Hence write down the range of $g(x)$. [1]

4 In the atmosphere, the air pressure P (Pascals) decreases with the height h (km) above sea level at a rate that is proportional to the pressure.

(i) Model this by a differential equation. [2]

At sea level the air pressure is 100 000 Pa.
At 1 km above sea level the air pressure is 88 000 Pa.

(ii) By solving the differential equation, find the air pressure at 400 m above sea level. [8]

5 **(a)** Use the substitution $u = x - 2$ to find

$$\int \frac{3x}{\sqrt{x-2}}\,dx$$

[7]

(b) Evaluate

$$\int_0^{\frac{\pi}{4}} 4x \cos 2x \, dx$$

[7]

6 A curve is defined by

$$x = t^4 - 6 \quad \text{and} \quad y = 2t^2 - 8t + 6$$

(i) Show that

$$\frac{dy}{dx} = \frac{t-2}{t^3}$$

[4]

(ii) Hence find the coordinates of the turning point and determine its nature. [9]

7 **(a)** Sketch the graph of

$$y = \sin^{-1} x$$

State the restricted domain of this function. [3]

(b) Solve the equation

$$\sin 2\theta = \cos\theta$$

for $-\pi \leqslant \theta \leqslant \pi$ [5]

(c) Prove the identity

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} \equiv \sec 2x$$

[7]