Rewarding Learning

ADVANCED
General Certificate of Education
January 2010

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

AMM21

MONDAY 1 FEBRUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 At time $t = 0$ seconds a body, P, has a velocity of $2\mathbf{i}\ \text{m s}^{-1}$ and is at a fixed point O.
P has a constant acceleration of $(5\mathbf{i} - \mathbf{j})\ \text{m s}^{-2}$

(i) Find the velocity of P when $t = 2$ [3]

(ii) Find the direction in which P is travelling at this time. [4]

2 A body of mass M kg is projected vertically downwards at $u\ \text{m s}^{-1}$
When it has fallen a distance x metres, its speed is $3u\ \text{m s}^{-1}$
Use the principle of conservation of mechanical energy to find x in terms of u and g. [7]

3 The displacement of a particle from a fixed point O at any time t seconds is given by

$$\mathbf{r} = (t^3\mathbf{i} + t^2\mathbf{j} + t\,\mathbf{k})\ \text{m}$$

(i) Find the velocity of the particle when $t = 3$ [3]

(ii) Find the speed of the particle when $t = 3$ [2]

(iii) Find an expression for the acceleration of the particle at time t. [2]

(iv) Explain why the acceleration of the particle is not constant. [1]

4 Take g to be 10 m s^{-2} in this question.

A car of mass 800 kg can **ascend** a hill inclined at an angle $\sin^{-1}\left(\frac{1}{64}\right)$ to the horizontal at a steady speed of 15 m s^{-1}
The resistance to motion is 275 N.

(i) Draw a diagram showing the external forces acting on the car. [2]

(ii) Show that the power developed by the car's engine is 6 kW. [6]

The car now travels **down** the same hill with the engine working at the same rate and against the same resistance.

(iii) Find the maximum speed of the car **down** the hill. [4]

5 A smooth ring of mass 0.1 kg is threaded onto a light inelastic string.
The ends of the string are attached to two fixed points A and B, where A is 0.4 m vertically above B.
The ring is made to move in horizontal circles with centre B.
The angle between the string and AB is 30° as shown in **Fig. 1** below.

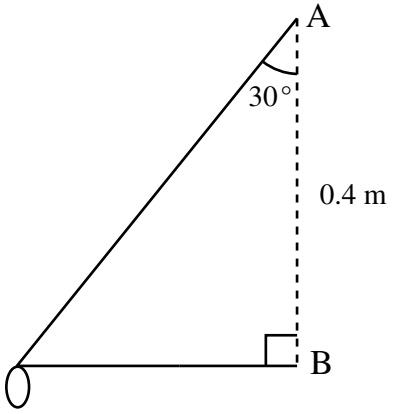

Fig. 1

(i) Draw a diagram showing the external forces acting on the ring. [2]

(ii) Find the tension in the string. [2]

(iii) Find the angular velocity at which the ring is moving. [7]

(iv) Find the time taken for the ring to complete one circle. [2]

6 A particle, P, is projected from horizontal ground with speed $u\,\mathrm{m\,s^{-1}}$ and at an angle θ to the horizontal.

(i) Show that the horizontal range of P is

$$\frac{u^2 \sin 2\theta}{g}$$ [6]

A golfer strikes a ball so that its initial velocity $u\,\mathrm{m\,s^{-1}}$ makes an angle of 27° with the horizontal. The range of the ball on the horizontal plane is 176.4 m.

(ii) Find u. [3]

(iii) Find the greatest height of the ball above the horizontal. [3]

7 A toy truck of mass 3 kg is travelling along a horizontal surface.
The truck's engine produces a forward force of 0.6 N.
The resistance to the motion of the truck is $9v$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the truck at any time t seconds.

(i) Show that the motion of the truck can be modelled by the differential equation.

$$\frac{dv}{dt} = 0.2 - 3v$$ [4]

The truck starts from rest.

(ii) Show that

$$t = \frac{1}{3} \ln \left| \frac{0.2}{0.2 - 3v} \right|$$ [8]

(iii) Find v when $t = 1$ [3]

(iv) State **one** further modelling assumption that you have made. [1]

THIS IS THE END OF THE QUESTION PAPER