GCE AS

Mathematics

Summer 2009

Mark Schemes

Issued: October 2009

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE) AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2009)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16 and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2009

Mathematics

Assessment Unit C1

assessing

Module C1: AS Core Mathematics 1

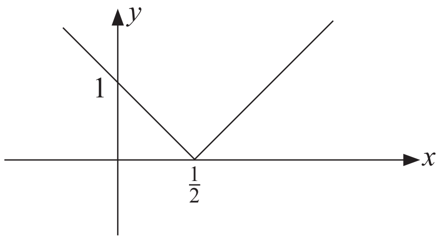
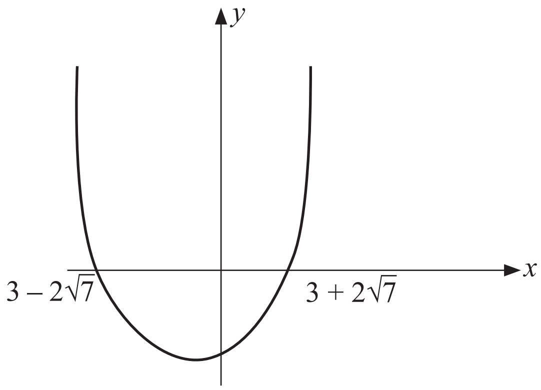
[AMC11]

FRIDAY 5 JUNE, AFTERNOON

MARK SCHEME

			AVAILABLE MARKS
1	**(i)** $m_{\text{AB}} = \frac{3 - -3}{-1 - 2} = \frac{6}{-3} = -2$	M1W1	
	$y - 3 = -2(x + 1)$	M1W1	
	$y + 2x = 1$		
	(ii) $4y - 2x = 6$		
	$y + 2x = 1$	M1	
	$5y = 7$		
	$y = 1.4$	W1	
	$x = -0.2$	W1	7
2	$4x - y + 2z = 13$		
	$\underline{2x + y + 2z = 6}$		
	$2x - 2y = 7$	M1W1	
	$4x - y + 2z = 13$		
	$\underline{4x - 4y - 2z = 6}$		
	$8x - 5y = 19$	MW1	
	$8x - 8y = 28$		
	$\underline{8x - 5y = 19}$		
	$-3y = 9$	M1	
	$y = -3$	W1	
	$x = \frac{1}{2}$	MW1	
	$z = 4$	MW1	7
3	**(a)** $\frac{dy}{dx} = 6x^2 - 4$	M1W1	
	$x = 1 \rightarrow m_t = 2$	MW1	
	$x = 1 \rightarrow y = 3$	MW1	
	$y - 3 = 2(x - 1)$	M1W1	
	$y = 2x + 1$		
	(b) **(i)** $\frac{dy}{dx} = 8x - x^{-2}$	MW2	
	(ii) $8x = \frac{1}{x^2}$	M1	
	$x^3 = \frac{1}{8}$		
	$x = \frac{1}{2}$	W1	
	$y = 4(\frac{1}{4}) + 2 = 3$	M1W1	
	$\frac{d^2y}{dx^2} = 8 + 2x^{-3}$	M1	
	$x = \frac{1}{2} \Rightarrow \frac{d^2y}{dx^2}$ is positive		
	$(\frac{1}{2}, 3)$ is a min tp	W1	14

				AVAILABLE MARKS
4	**(i)**	$f(l) = 9 - 9 - 1 + 1 = 0 \quad \therefore (x - 1)$ is a factor	M1W1	
	(ii)	$\begin{array}{r} 9x^2 - 1 \\ x - 1\overline{)9x^3 - 9x^2 - x + 1} \\ \underline{9x^3 - 9x^2} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$	M1W1	
		$(x - 1)(9x^2 - 1) = (x - 1)(3x - 1)(3x + 1)$	MW1	
	(iii)	$\dfrac{(x - 1)(3x - 1)(3x + 1)}{(3x - 1)} \div \dfrac{(x - 1)}{4}$	M1	
		$\dfrac{(x - 1)(3x - 1)(3x + 1)}{(3x - 1)} \times \dfrac{4}{(x - 1)}$	M1	
		$4(3x + 1)$	W2	9
5	**(i)**	$C = 80$ initial temperature 80 °C	MW1	
	(ii)	$80 - 10t + \frac{1}{2}t^2 = 50$ $t^2 - 20t + 60 = 0$	M1	
		$t = \dfrac{20 \pm \sqrt{400 - 240}}{2}$	M1	
		$t = \dfrac{20 \pm \sqrt{160}}{2} = 10 \pm 2\sqrt{10}$	W1	
		$t = 10 - 2\sqrt{10}$ only	W1	
	(iii)	$\dfrac{dC}{dt} = -10 + t$	M1W1	
		$t = 3 \Rightarrow \dfrac{dC}{dt} = -7$	M1W1	9
6	**(i)**	$3xy = 66$	M1	
		$y = \dfrac{22}{x}$	W1	
	(ii)	$SA = 2xy + 6x + 6y = 101$	M1MW1	
	(iii)	$44 + 6x + 6\left(\dfrac{22}{x}\right) = 101$	MW1	
		$6x + \dfrac{132}{x} - 57 = 0$		
		$6x^2 - 57x + 132 = 0$ $2x^2 - 19x + 44 = 0$	MW1	
		$(2x - 11)(x - 4) = 0$	M1	
		$x = 5.5$ or $x = 4$ $y = 4$ or $y = 5.5$	W1	
		Dimensions 4 cm × 5.5 cm × 3 cm	W1	9

				AVAILABLE MARKS
7	**(a)**	$\frac{5(1-\sqrt{3})-(\sqrt{3}+1)}{(\sqrt{3}+1)(1-\sqrt{3})}$	M1 MW1	
		$\frac{4-6\sqrt{3}}{-2}=3\sqrt{3}-2$	MW2	
	(b)	$\frac{(5)^x}{(5^2)^{x-1}}=5^{\frac{1}{2}}$	M1W1	
		$5^{x-2(x-1)}=5^{\frac{1}{2}}$	M1W1	
		$5^{2-x}=5^{\frac{1}{2}}$		
		$2-x=\frac{1}{2}$	M1	
		$x=1\frac{1}{2}$	W1	
	(c)		M1W1	12
8		$b^2-4ac<0$		
		$(3-k)^2-28<0$	M2W1	
		$k^2-6k-19<0$		
		If $k^2-6k-19=0$	W1	
		Then $k=\frac{6\pm\sqrt{36+76}}{2}$	M1	
		$k=\frac{6\pm\sqrt{112}}{2}=3\pm2\sqrt{7}$	W1	
			M1	
		$3-2\sqrt{7}<k<3+2\sqrt{7}$	W1	8
			Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2009

Mathematics

Assessment Unit C2
assessing
Module C2: Core Mathematics 2

[AMC21]

FRIDAY 22 MAY, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

AVAILABLE MARKS

1 **(a)** **(i)** $3x^3 + 2x + 4x^{-2}$ MW1

(ii) $\int 3x^3 + 2x + 4x^{-2}\,dx$

$\frac{3x^4}{4} + x^2 - 4x^{-1} + c$ MW4

(b) $h = \frac{1}{5}$ MW1

x	y
0	4.0000
0.2	3.8462
0.4	3.4483
0.6	2.9412
0.8	2.4390
1	2.0000

x values, y values M1W2

$\int \approx \frac{1}{2}h(y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5)$

$= \frac{1}{2} \times 0.2(4.0000 + 2(3.8462 + 3.4483 + 2.9412 + 2.4390) + 2.0000)$ M1

$= 3.13(3.13494)$ MW1 11

2 **(a)** **(i)** $u_2 = \frac{2}{3}u_1 = \frac{2}{3}(1) = \frac{2}{3}$ MW1

$u_3 = \frac{2}{3}(\frac{2}{3}) = \frac{4}{9}$ MW1

$u_4 = \frac{2}{3}(\frac{4}{9}) = \frac{8}{27}$ MW1

(ii) convergent MW1

(b) **(i)** $\frac{2}{3}$ MW1

(ii) $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{2}{3}}$ M1

$S_\infty = 3$ W1 7

3 $(1 + nx)^{10}$

$= 1 + 10nx + \dfrac{10.9n^2x^2}{2.1} \ldots.$ M1W1

$= 1 + 10nx + 45n^2x^2 \ldots.$ W1

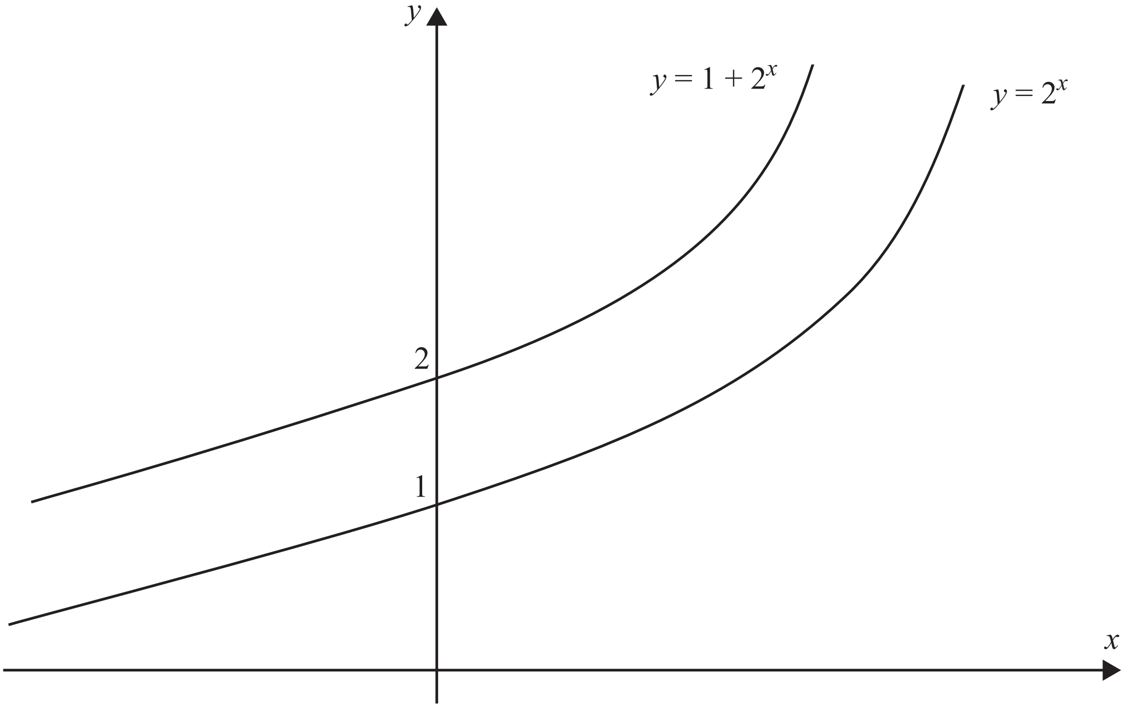
Hence, $45n^2 = 3(10n)$ M1W1

$45n^2 - 30n = 0$

$15n(3n - 2) = 0$

$n = \frac{2}{3} \quad n \neq 0$ W1 6

4 **(i)**



exponential shape, (0, 1) M1W1

shift up parallel to x-axis, (0, 2) M1W1

(ii) $1 + 2^x = 6$

$2^x = 5$ MW1

$\log_{10}2^x = \log_{10}5$ M1

$x\log_{10}2 = \log_{10}5$ M1

$x = \dfrac{\log_{10}5}{\log_{10}2}$ W1 8

$x = 2.32$

				AVAILABLE MARKS
5	**(a)**	$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$		
		L.H.S. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$	M1W1	
		$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$	MW1M1	
		$= \frac{1}{\sin\theta\ \cos\theta}$	W1	
	(b)	$\sin^2 x = \frac{1}{4}$		
		$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$	MW2	
		$x = 30°$ or $x = -30°$	MW2	
	(c)	$\cos 2x = 0.4$		
		$2x = 1.159, 5.124$	M1W2	
		$x = 0.580, 2.56$	MW1	13
6	**(i)**	Area $= \int_0^1 (4 + x^2)\, \mathrm{d}x$	M2W1	
		$= \left[4x + \frac{x^3}{3}\right]_0^1$	MW1	
		$= [4 + \frac{1}{3}] - [0 - 0]$		
		$= 4\frac{1}{3}$	W1	
	(ii)	y coord $= 5 = 4 + x^2 \ldots . x = 1$ or -1	MW1	
		Area of rectangle $= 5 \times 1 = 5$	MW1	
		Area required $= 2(5 - 4\frac{1}{3}) = \frac{4}{3}$	M1MW1	9

		AVAILABLE MARKS
7 **(i)** Gradient $AC = \frac{1-(-1)}{2-(-1)} = \frac{2}{3}$	M1W1	
(ii) Gradient $BC = -\frac{3}{2}$	MW1	
Gradient $BC = \frac{(k+5)-(-1)}{k-(-1)} = \frac{k+6}{k+1} = \frac{-3}{2}$	M1W1	
$2(k+6) = -3(k+1)$		
$2k+12 = -3k-3$		
$5k = -15$		
$k = -3$	MW1	
(iii) Coordinates of B = (–3, 2)		
Centre $\frac{(2+(-3))}{2}, \frac{(1+2)}{2} = (-0.5, 1.5)$	MW2	
Radius $= \sqrt{(2--0.5)^2 + (1-1.5)^2} = \sqrt{(2.5)^2 + (-0.5)^2} = \sqrt{6.5}$	M1W1	
Equation $(x+0.5)^2 + (y-1.5)^2 = 6.5$	MW1	11
8 **(i)** Area $= \frac{1}{2}r^2 x$	MW1	
(ii) Area $= \frac{1}{2}r^2 \sin x$	MW1	
(iii) Area of major segment $= \frac{1}{2}r^2(2\pi - x) + \frac{1}{2}r^2 \sin x$	M1W1	
Area of minor segment $= \frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x$	M1W1	
Hence, $\frac{1}{2}r^2(2\pi - x) + \frac{1}{2}r^2 \sin x = 5(\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x)$	M1W1	
$(2\pi - x) + \sin x = 5(x - \sin x)$		
$2\pi - x + \sin x = 5x - 5 \sin x$		
$6 \sin x = 6x - 2\pi$		
$\sin x = x - \frac{\pi}{3}$	MW2	10
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2009

Mathematics

Assessment Unit F1
assessing
Module FP1: Further Pure Mathematics 1

[AMF11]

TUESDAY 23 JUNE, MORNING

MARK
SCHEME

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				AVAILABLE MARKS
1	**(a)**	Rotation of 45° anticlockwise about the origin	MW2	
	(b)	$\begin{pmatrix}-1 & 1\\ 6 & -2\end{pmatrix}\begin{pmatrix}t\\ mt\end{pmatrix}=\begin{pmatrix}x\\ mx\end{pmatrix}$	M1	
		Hence $\begin{array}{c}-t+mt=x\\ 6t-2mt=mx\end{array}$	MW1	
		Dividing $\Rightarrow \dfrac{6-2m}{-1+m}=\dfrac{m}{1}$	M1	
		$\Rightarrow 6-2m=-m+m^2$	W1	
		$\Rightarrow m^2+m-6=0$		
		$\Rightarrow (m+3)(m-2)=0$		
		$\Rightarrow m=-3, 2$	W1	
		Hence the equations of the lines are $y=-3x$, $y=2x$	W1	8
2	**(i)**	$\mathbf{M}^2=\begin{pmatrix}-1 & 1\\ -2 & 4\end{pmatrix}\begin{pmatrix}-1 & 1\\ -2 & 4\end{pmatrix}$	M1	
		$=\begin{pmatrix}-1 & 3\\ -6 & 14\end{pmatrix}$	W1	
		$3\mathbf{M}+2\mathbf{I}=\begin{pmatrix}-3 & 3\\ -6 & 12\end{pmatrix}+\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$	M1	
		$=\begin{pmatrix}-1 & 3\\ -6 & 14\end{pmatrix}$	W1	
		Hence $\mathbf{M}^2=3\mathbf{M}+2\mathbf{I}$		
	(ii)	$\mathbf{M}^4=(\mathbf{M}^2)^2=(3\mathbf{M}+2\mathbf{I})^2$	M1	
		$=9\mathbf{M}^2+12\mathbf{M}+4\mathbf{I}$	W1	
		$=9\,(3\mathbf{M}+2\mathbf{I})+12\mathbf{M}+4\mathbf{I}$	M1	
		$=39\mathbf{M}+22\mathbf{I}$	W1	8

AVAILABLE MARKS

3 **(i)** $[(a, b)*(c, d)]*(e, f) = (ad + bc, bd)*(e, f)$ M1

$= (adf + bfc + bde, bdf)$ W1

$(a, b)*[(c, d)*(e, f)] = (a, b)*(cf + de, df)$ M1

$= (adf + bcf + bde, bdf)$ W1

Hence the associative law holds.

(ii) $(a, b)*(c, d) = (a, b)$ if (c, d) is the identity

Hence $ad + bc = a$ and $bd = b$ M1M1

$\Rightarrow b(d - 1) = 0$

$\Rightarrow d = 1$ since $b \neq 0$ W1

$\Rightarrow a + bc = a$ since $d = 1$

$\Rightarrow c = 0$ since $b \neq 0$ W1

Also $(0, 1)*(a, b) = (a, b)$

Hence the identity element is $(0, 1)$

(iii) $(a, b)*(c, d) = (0, 1)$ if (c, d) is the inverse of (a, b)

Hence $ad + bc = 0$ and $bd = 1$ M1

$\Rightarrow d = \frac{1}{b}$ W1

$\Rightarrow \frac{a}{b} + bc = 0$

$\Rightarrow c = -\frac{a}{b^2}$ W1

Hence the inverse is $\left(-\frac{a}{b^2}, \frac{1}{b}\right)$ 11

AVAILABLE MARKS

4 **(i)** det $(\mathbf{N} - \lambda\mathbf{I}) = 0$ M1

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & -6 \\ 3 & 1-\lambda & 4 \\ -1 & 0 & 1-\lambda \end{vmatrix} = 0$$ M1

$\Rightarrow (2-\lambda)\{(1-\lambda)^2 - 0\} - 6\{0 + (1-\lambda)\} = 0$ M1

$\Rightarrow (1-\lambda)\{(2-\lambda)(1-\lambda) - 6\} = 0$

$\Rightarrow (1-\lambda)(2 - 3\lambda + \lambda^2 - 6) = 0$

$\Rightarrow (1-\lambda)(\lambda^2 - 3\lambda - 4) = 0$ W1

$\Rightarrow (1-\lambda)(\lambda - 4)(\lambda + 1) = 0$

$\Rightarrow \lambda = 1, 4, -1$ W3

(ii) $$\begin{pmatrix} 2 & 0 & -6 \\ 3 & 1 & 4 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ M1

$2x - 6z = 4x$

$\Rightarrow 3x + y + 4z \;\; 4y$ M1

$-x + z = 4z$

① and ③ $\Rightarrow x = -3z$

② $\Rightarrow -9z - 3y + 4z = 0$

$\Rightarrow y = -\frac{5}{3}z$

Hence an eigenvector is $\begin{pmatrix} -9 \\ -5 \\ 3 \end{pmatrix}$ W1

A unit eigenvector is therefore $\frac{1}{\sqrt{115}}\begin{pmatrix} -9 \\ -5 \\ 3 \end{pmatrix}$ W1 11

5 **(a)** $(a + b\text{i})^2 = 21 - 20\text{i}$ M1

$\Rightarrow a^2 + 2ab\text{i} - b^2 = 21 - 20\text{i}$

Hence $a^2 - b^2 = 21$ and $2ab = -20$ M1W1

$\Rightarrow b = -\dfrac{10}{a}$ MW1

Substituting into equation ① gives

$a^2 - \dfrac{100}{a^2} = 21$ M1

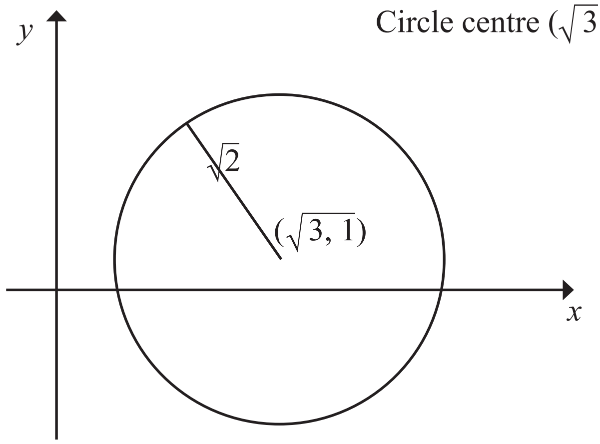
$\Rightarrow a^4 - 21a^2 - 100 = 0$ W1

$\Rightarrow (a^2 - 25)(a^2 + 4) = 0$

$\Rightarrow a = \pm 5$ W1

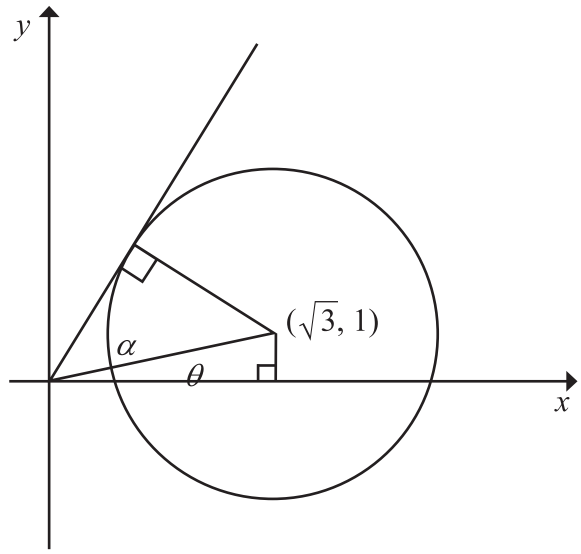
$\Rightarrow b = \dfrac{-10}{\pm 5} = \mp 2$ W1

(b) **(i)** Circle centre $(\sqrt{3}, 1)$ and radius $\sqrt{2}$ MW2

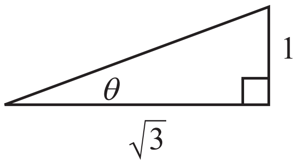


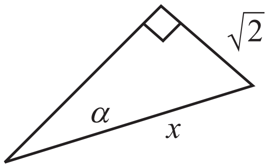
MW1

(ii)



$\arg z \leq \alpha + \theta$ M1





$\tan \theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$ MW1

$x^2 = 3 + 1 = 4$

$\Rightarrow x = 2$ MW1

$\sin \alpha = \frac{\sqrt{2}}{2}$

$\Rightarrow \alpha = \frac{\pi}{4}$ MW1

Hence $\arg z \leq \frac{\pi}{4} + \frac{\pi}{6}$

$\Rightarrow \arg z \leq \frac{5\pi}{12}$ W1 16

				AVAILABLE MARKS
6	**(i)**	C_1 has centre $(-1, 7)$	MW1	
		Gradient of radius $= \dfrac{7-6}{-1-2} = \dfrac{1}{-3}$	M1W1	
		Gradient of tangent = 3	MW1	
		Equation of tangent is $y - 6 = 3(x - 2)$	M1	
		which gives $y = 3x$	W1	
	(ii)	Let equation of tangent be $y = mx$		
		Then $x^2 + m^2x^2 + 2x - 14mx + 40 = 0$	M1	
		$\Rightarrow x^2(1 + m^2) + x(2 - 14m) + 40 = 0$	W1	
		If line is a tangent then $b^2 - 4ac = 0$		
		$\Rightarrow (2 - 14m)^2 = 4(40)(1 + m^2)$	M1	
		$\Rightarrow 4 - 56m + 196m^2 = 160 + 160m^2$	W1	
		$\Rightarrow 36m^2 - 56m - 156 = 0$		
		$\Rightarrow 9m^2 - 14m - 39 = 0$	W1	
		$\Rightarrow (m - 3)(9m + 13) = 0$		
		$m = 3, -\dfrac{13}{9}$	W1	
		Therefore the other tangent has equation $y = -\dfrac{13}{9}x$	W1	
	(iii)	$x^2 + y^2 + 2x - 14y + 40 = 0$		
		$x^2 + y^2 - 10x - 8y + 16 = 0$		
		Subtract to give $12x - 6y + 24 = 0$	M1	
		$\Rightarrow y = 2x + 4$	W1	
		Substitute into equation ① to give		
		$x^2 + (2x + 4)^2 + 2x - 14(2x + 4) + 40 = 0$	M1	
		$\Rightarrow x^2 + 4x^2 + 16x + 16 + 2x - 28x - 56 + 40 = 0$		
		$\Rightarrow 5x^2 - 10x = 0$	W1	
		$\Rightarrow 5x(x - 2) = 0$		
		$\Rightarrow x = 0, 2$	W2	
		$\Rightarrow y = 4, 8$	W2	
		Therefore the points of intersection are (0, 4) and (2, 8)		21
			Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2009

Mathematics

Assessment Unit M1
assessing
Module M1: Mechanics 1

[AMM11]

FRIDAY 15 MAY, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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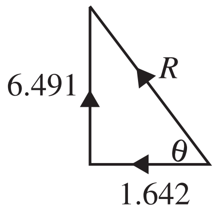
AVAILABLE MARKS

M1

1 $\rightarrow 8 - 15\cos 50° = -1.642\text{ N}$ M1W1

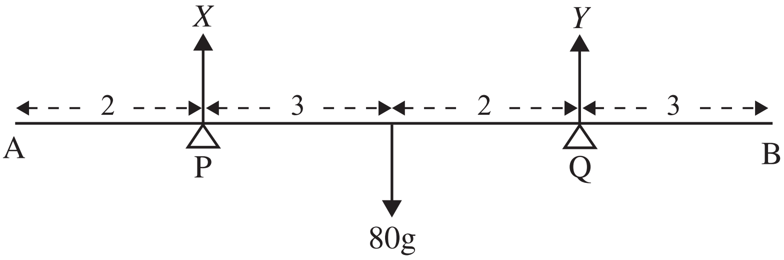
$\uparrow$ $15\sin 50° - 5 = 6.491\text{ N}$ MW1



$R = \sqrt{6.491^2 + 1.642^2} = 6.70\text{ N}$ M1W1

$\theta = \tan^{-1}\dfrac{6.491}{1.642} = 75.8°$ M1W1 8

2 **(i)**



MW2

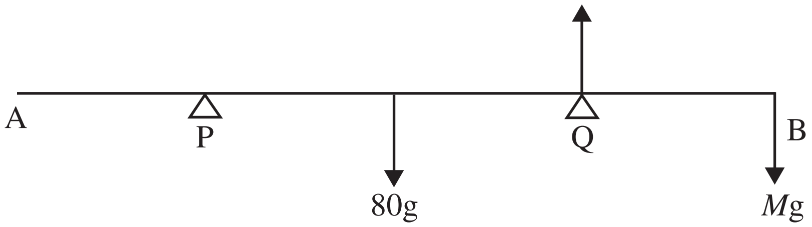
(ii) $M(\text{P})\ 80\text{g} \times 3 = Y \times 5$ M2W1

$48\text{gN} = Y$ W1

$X + Y = 80\text{g}$ M1

$\uparrow$ $X = 32\text{gN}$ W1

(iii)



Reaction at P = 0 when about to tilt M1

$M(\text{Q})\ 3M\text{g} = 80\text{g} \times 2$ M1W1

$M = \dfrac{160}{3}\text{ kg}$ W1 12

AVAILABLE MARKS

3 **(i)** $u = 14\,\text{m s}^{-1}$

$a = -9.8\,\text{m s}^{-2}$

$v = 0$ MW1

$s = h$

By $v^2 = u^2 + 2as$ M1W1

$0 = 14^2 + 2(-9.8)h$

$h = 10\,\text{m}$ W1

(ii) $u = 14$

$a = 9.8$

$s = 8.4$

$t = ?$

By $s = ut + \frac{1}{2}at^2$

$8.4 = 14t + \frac{1}{2}(-9.8)t^2$ M1W1

$8.4 = 14t - 4.9t^2$

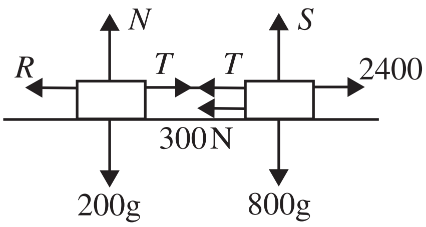
$7t^2 - 20t + 12 = 0$

$(7t - 6)(t - 2) = 0$

$t = \frac{6}{7}$ or $t = 2$ W1

So above 8.4 m for $2 - \frac{6}{7} = \frac{8}{7}$ s M1W1 9

4 **(i)**



MW2

(ii) Using $F = ma$

car $2400 - T - 300 = 800 \times 2$ M1W1

trailer $T - R = 200 \times 2$ M1W1

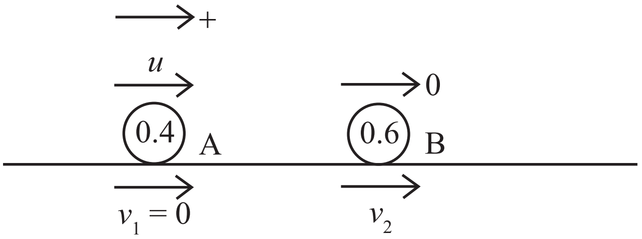
system $2400 - 300 - R = 2000$ M1

$R = 100\,\text{N}$ W1

$T = 400 + R$ W1 9

$T = 500\,\text{N}$

5



(i) By conservation of momentum

$0.4u + 0.6 \times 0 = 0.4 \times 0 + 0.6 \times v_2$ M2W1

$\frac{0.4u}{0.6} = v_2$

$v_2 = \frac{2u}{3}\ \text{m s}^{-1}$ W1

(ii) Using I = increase in momentum of A

$I = 0.4 \times v_1 - 0.4u$ M1

$I = -0.4u$ N s W1W1

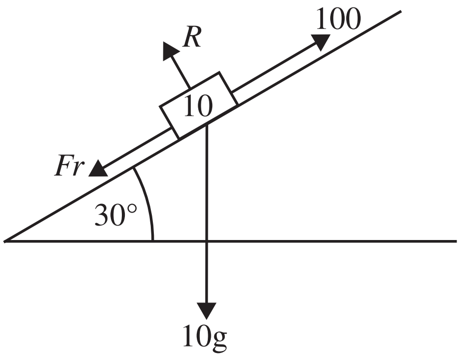
or $I = 0.6\,v_2 - 0.6 \times 0$ M1

$= 0.6 \times \frac{2u}{3}$ W1

$I = 0.4u$ N s

so impulse on A is $-I = -0.4u$ N s W1 7

6 **(i)**



MW2

(ii) Using $F = ma$

↖ $R - 10g\cos 30° = 0$

$R = 10g\cos 30°$ MW1

$F = \mu R$

$= 0.3 \times 10g\cos 30°$ M1

$= 25.46$

↗ $100 - \mu R - 10g\sin 30° = 10a$ M1

$100 - 0.3 \times 10g\cos 30° - 10g\sin 30° = 10a$

$100 - 25.46 - 49 = 10a$ W2

$25.54 = 10a$

$2.55\ \text{m s}^{-2} = a$ W1 8

			AVAILABLE MARKS
7	**(i)** $v = t^2 - 5t + 6$		
	$v = 0 \Rightarrow t^2 - 5t + 6 = 0$	M1	
	$(t-2)(t-3) = 0$		
	$t = 2s$ or $3s$	W2	
	(ii) $a = \frac{\mathrm{d}v}{\mathrm{d}t} = (2t - 5)\ \mathrm{m\,s^{-2}}$	M1W1	
	(iii) $s = \int v\ \mathrm{d}t$		
	$= \int t^2 - 5t + 6\ \mathrm{d}t$	M1	
	$s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$	W1	
	$t = 6, s = 0$ so		
	$0 = \frac{6^3}{3} - 5 \times \frac{6^2}{2} = 6 \times 6 + c$		
	$0 = 72 - 90 + 36 + c$		
	$c = -18$	M1W1	
	$s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t - 18$		
	(iv) $t = 0 \quad s = -18$		
	$t = 2 \quad s = \frac{2^3}{3} - \frac{5(2)^2}{2} + 6(2) - 18 = -13\frac{1}{3}$	MW2	
	$t = 3 \quad s = \frac{(3)^3}{3} - \frac{5(3)^2}{2} + 6(3) - 18 = -13\frac{1}{2}$		
	Distance travelled $4\frac{2}{3} + \frac{1}{6} = 4\frac{5}{6}$ m	M1W1	13

8 Assume police car catches motorcyclist when $t = T$.

Motorcyclist $u = 30\,\mathrm{m\,s^{-1}}$ MW1

$a = 0.2\,\mathrm{m\,s^{-2}}$

$s = d$

$t = T$

Using $s = ut + \frac{1}{2}at^2$ M1

$d = 30T + \frac{1}{2}(0.2)T^2$ W1

$d = 30T + 0.1T^2$

Police car $u = 0$ MW1

$a = 1\,\mathrm{m\,s^{-2}}$

$t = T - 5$ MW1

$s = d$

Using $s = ut + \frac{1}{2}at^2$

$d = 0(T-5) + \frac{1}{2}(1)(T-5)^2$ W1

Hence $30T + 0.1T^2 = \frac{1}{2}(T^2 - 10T + 25)$ M1

$30T + 0.1T^2 = 0.5T^2 - 5T + 12.5$ W1

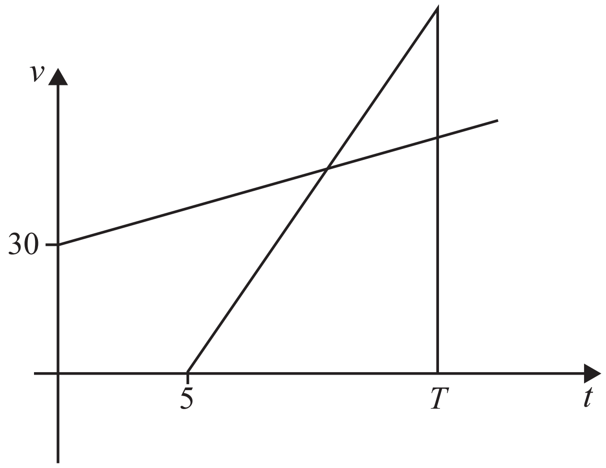
$0.4T^2 - 35T + 12.5 = 0$

$4T^2 - 350T + 125 = 0$

$T = 87.1$ or 0.359 (not valid)

Hence $T = 87.1\mathrm{s}$ W1

Alternative solution



Draw level when $t = T =$

Area under graphs equal

$\frac{1}{2}(T-5)^2 = \frac{1}{2}(30 + 0.2T + 30)\ T$ MW1M3W2

$T^2 - 10T + 25 = 60T + 0.2T^2$ W1

$0.8T^2 - 70T + 25 = 0$ W1

$T = 87.1$ or 2.9 (not valid)

Hence $T = 87.1$s W1 9

Total **75**

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Assessment Unit S1
assessing
Module S1: Statistics 1

[AMS11]

MONDAY 1 JUNE, MORNING

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				AVAILABLE MARKS
1	Mid-values 5, 15, 25, 35, 45		MW1	
	Summary values from calculator			
	$n = 75 \quad \sum fx = 1815 \quad \sum fx^2 = 51275$		M1	
	$\bar{x} = 24.2$		W1	
	$\sigma_{n-1} = 9.97$ (3 s.f.)		M1W1	5
2	**(i)**	The individuals are independent	M1	
		Either left-handed or not (2 outcomes)	M1	
	(ii)	Let X be r.v. "No. of left-handed customers" then $X \sim \text{Bin}(8, 0.14)$		
		$P(X = 1) = \binom{8}{1}(0.14)(0.86)^7$	M1	
		$= 0.390$ (3 s.f.)	W1	
	(iii)	$P(X \geqslant 3) = 1 - P(X < 3)$		
		$= 1 - [P(X = 0) + (X = 1) + P(X = 2)$	M1	
		$P(X = 2) = \binom{8}{2}(0.14)^2\,(0.86)^6 = 0.2220$	MW1	
		$P(X = 0) = \binom{8}{0}(0.14)^0\,(0.86)^8 = 0.2992$	MW1	
		$P(X \geqslant 3) = 1 - [0.2992 + 0.3897 + 0.2220]$		
		$= 0.0891$ (3 s.f.)	W1	8

				AVAILABLE MARKS
3	**(i)**	$8k = 1$	M1	
		$k = 0.125$	W1	
	(ii)	Symmetrically mid-way between 5 and 12	M1	
	(iii)	$E(X^2) = 0.125(5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2)$	M1	
		$= 77.5$	W1	
		$\text{Var}(X) = E(X^2) - [E(X)]^2$	M1	
		$= 77.5 - 8.5^2$		
		$= 5.25$	W1	
	(iv)	$E(Y) = 2 \times 8.5 - 5 = 12$	MW1	
		$\text{Var}(Y) = 2^2 \times 5.25 = 21$	M1W1	10
4		X is r.v. "No. of goals scored during a match"		
	(i)	$\lambda = \frac{21}{35} = 0.6 \quad X \sim \text{Po}(0.6)$	MW1	
		$\text{P(scores)} = P(X \geqslant 1) = 1 - P(X = 0)$	M1	
		$P(X = 0) = \frac{e^{-0.6}\, 0.6^0}{0!}$		
		$= 0.5488\ldots.$	M1	
		$\text{P(scores)} = 0.451188\ldots. = 0.451$ (3 s.f.)	W1	
	(ii)	$P(X = 1) = \frac{e^{-0.6} \times 0.6^1}{1!} = 0.6e^{-0.6} = 0.32928$	MW1	
		$P(X = 2) = \frac{e^{-0.6} \times 0.6^2}{2!} = 0.18e^{-0.6} = 0.09878$	MW1	
		$P(X = 1 \text{ or } 2) = 0.428$ (3 s.f.)	W1	
	(iii)	$P(X \geqslant 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$	M1	
		$= 1 - (0.5488 + 0.32928 + 0.09878)$		
		$= 0.02311\ldots.$	W1	
		$E(\text{Bonus}) = 1000 \times P(X = 1 \text{ or } 2) + 5000 \times P(X \geqslant 3)$	M1	
		$= 1000 \times 0.42807 + 5000 \times 0.02311\ldots.$	W1	
		= £543.62	W1	12

5 **(i)** Let X be r.v. mass of student, $X \sim \mathrm{N}(\mu, 12^2)$

$$\mathrm{P}(X < 111.74) = 0.95 \Rightarrow \mathrm{P}\left(Z < \frac{111.74 - \mu}{12}\right) = 0.95$$ M1

$$\Rightarrow \frac{111.74 - \mu}{12} = \Phi^{-1}(0.95)$$ M1

$$\frac{111.74 - \mu}{12} = 1.645$$ W1

$$\Rightarrow \mu = 111.74 - 12 \times 1.645 = 92$$ W1

(ii) $$\mathrm{P}(\mathrm{X} < 89) = \mathrm{P}\left(Z < \frac{89 - 92}{12}\right) = \mathrm{P}(Z < -0.25)$$ W1

$$= 1 - \Phi(0.25)$$ M1

$$= 1 - 0.5987$$ W1

$$= 0.4013 \quad 0.401 \text{ (3 s.f.)}$$ W1

(iii) $$\mathrm{P}(89 < X < 98) = \mathrm{P}\left(\frac{89 - 92}{12} < Z < \frac{98 - 92}{12}\right)$$

$$= \mathrm{P}(-0.25 < Z < 0.5)$$ W1

$$= \Phi(0.5) - \Phi(0.25)$$ M1

$$= 0.6915 - 0.4013 = 0.2902 = 0.290 \text{ (3 s.f.)}$$ W1,W1

(iv) $$\mathrm{P}(X < W) = 0.8$$

$$\Rightarrow \mathrm{P}\left(Z < \frac{W - 92}{12}\right) = 0.8$$ M1

$$\Rightarrow \Phi\left(\frac{W - 92}{12}\right) = 0.8$$

$$\frac{W - 92}{12} = \Phi^{-1}(0.8)$$ MW1

$$= 0.842$$ W1

$$W = 12 \times 0.842 + 92 = 102.104$$ W1 16

$$102 \text{ (3 s.f.)}$$

AVAILABLE MARKS

6 **(i)** f(2) = 2k MW1

(ii) Area = 1 M1

$\frac{1}{2} \times 3 \times 2k = 1$ MW1

$\Rightarrow k = \frac{1}{3}$ W1

(iii) $P(1 \leqslant X \leqslant 3) = 1 - P(X \leqslant 1)$ M1

$= 1 - \frac{1}{2} \times 1 \times \frac{1}{3}$ MW1

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$ W1

(iv) Let m = median of X $[m < 2]$ M1

$f(m) = \frac{1}{3}m$ W1

$P(X < \times m) = \frac{1}{2}$ M1

$\therefore \frac{1}{2} \times m \times \frac{1}{3} m = \frac{1}{2}$ W1

$m^2 = 3$

$m = \sqrt{3}$ W1 12

7 **(i)** $P(F \cap C) = P(F) \times P(C|F)$ M1

$0.082 = P(F) \times 0.2$

$P(F) = \dfrac{0.082}{0.2} = 0.41$ W1

(ii) $P(F \cap C) = P(C) \times P(F|C)$ M1

$0.082 = (C) \times 0.25$

$P(C) = \dfrac{0.082}{0.25} = 0.328$ W1

(iii) $P(F \cup C) = P(F) + P(C) - P(F \cap C)$ M1

$= 0.41 + 0.328 - 0.082 = 0.656$ W1

$P(\overline{F} \cap \overline{C}) = 1 - 0.656 = 0.344$ MW1

(iv) $P(F|\overline{C}) = \dfrac{P(F \cap \overline{C})}{P(\overline{C})}$ M1

$P(\overline{C}) = 1 - P(\overline{C}) = 1 - 0.328 = 0.672$ MW1

$P(F \cap \overline{C}) = P(F) - P(F \cap C) = 0.41 - 0.082 = 0.328$ MW1

$P(F|\overline{C}) = \dfrac{0.328}{0.672} = 0.488$ (3 s.f.) W2 12

Total **75**