Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

AMM21

[AMM21]

THURSDAY 11 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answer should be given to three significant figures unless otherwise stated.

1 A particle of mass 5 kg is acted on by three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ newtons, where

$$\mathbf{F}_1 = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{F}_2 = 5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{F}_3 = p\mathbf{i} + q\mathbf{j} - 3\mathbf{k}$$

The particle is moving with acceleration $(\mathbf{i} + \mathbf{j})$ m s^{-2}

(i) Find p and q. [7]

When the particle is at the origin O, it has an initial velocity of $(\mathbf{i} + 2\mathbf{k})$ m s^{-1}

(ii) Find its velocity after 3 seconds. [3]

(iii) Find its displacement from O after 6 seconds. [3]

2 **Fig. 1** below shows a skier of mass 80 kg subject to a resistance, R, as he descends a slope. He starts from rest at a point, A, 800 m above sea level and skis to the bottom of the slope, B, which is 500 m above sea level.
His velocity at B is 15 m s^{-1}

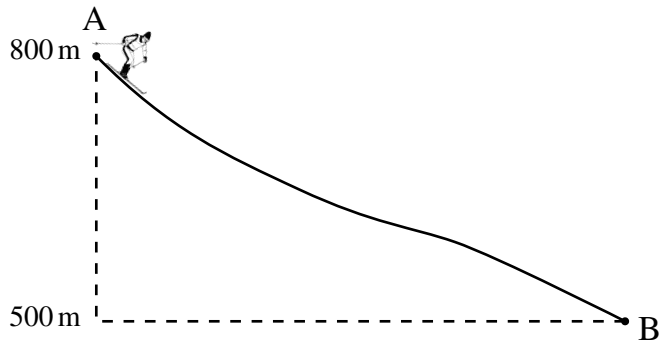

Fig. 1

(i) Find the increase in the kinetic energy of the skier. [3]

(ii) Find the work done by gravity. [2]

(iii) **Hence**, using the Work–Energy Principle, find the work done by R. [5]

(iv) State one modelling assumption you have made in answering this question. [1]

3 At time t seconds, a particle has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ given by

$$\mathbf{v} = 3t\mathbf{i} - 3t\mathbf{j} + 3\mathbf{k}$$

(i) Find the **speed** of the particle when $t = 2$ [3]

At $t = 0$ the particle has displacement $(\mathbf{i} + 3\mathbf{j})$ metres relative to an origin O.

(ii) Find the displacement of the particle from O at time $t = 4$ [5]

4 Ann and her bicycle have a total mass of 60 kg.
She cycles at a constant speed of $8\,\mathrm{m\,s^{-1}}$ along a straight horizontal road against a resisting force S newtons.
Ann works at a constant rate of 500 W.

(i) Find S. [4]

Ann now **ascends** a hill inclined at $\sin^{-1}\frac{1}{7}$ to the horizontal.
She continues to work at 500 W and the resisting force remains the same.

(ii) Find her acceleration when she is ascending the hill with a speed of $2\,\mathrm{m\,s^{-1}}$ [6]

5 **Fig. 2** below shows two particles P and Q attached by a light inextensible string.
The string passes through a smooth hole, O, in a smooth horizontal surface.
Q, mass 5 kg, hangs vertically below O and remains at rest.
P, mass 2 kg, moves in horizontal circles, centre O, on the surface.

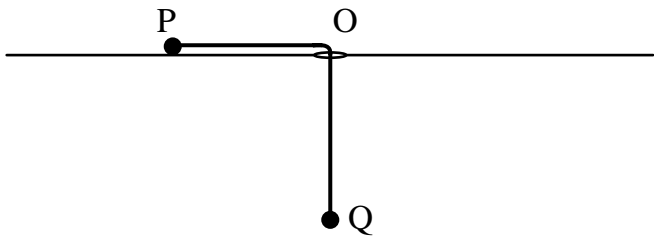

Fig. 2

(i) Draw a diagram showing all the external forces acting on P and Q. [2]

(ii) Find the tension in the string. [2]

P moves with angular speed $10\,\text{rad}\,\text{s}^{-1}$

(iii) Find the radius of the circle. [5]

6 An ice puck of mass 0.2 kg has an initial speed of $25\,\text{m}\,\text{s}^{-1}$ along an icy horizontal surface.
During its time of motion, a horizontal resistance of magnitude $0.005v^2$ N acts to oppose its motion.
At time t seconds, the puck moves with velocity $v\,\text{m}\,\text{s}^{-1}$
Model the puck as a particle.

(i) Show that the equation of motion of the particle can be modelled by

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.025v^2$$ [3]

(ii) Find the speed of the puck when $t = 2$ [8]

7 A particle is projected from a point O on horizontal ground with a speed of $u\,\text{m s}^{-1}$ and at an angle θ above the horizontal, as shown in **Fig. 3** below.

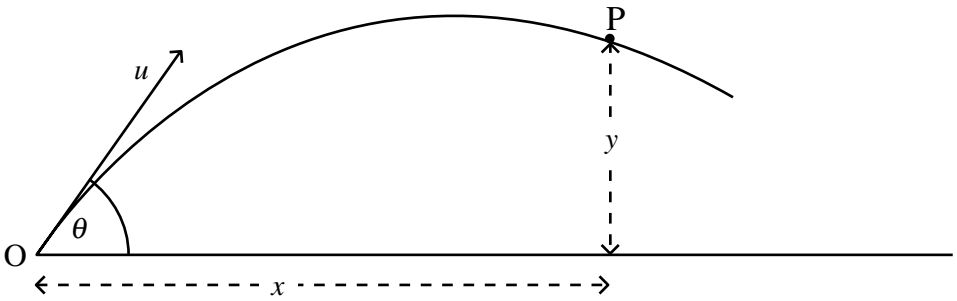

Fig. 3

After t seconds, the particle passes through a point P.
P is x metres horizontally and y metres vertically from O.

(i) Show that the time taken for the particle to reach P is

$$t = \frac{x}{u\cos\theta}$$ [3]

(ii) Hence, show that the particle follows a path whose equation is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$ [4]

The crossbar of a set of rugby posts on a horizontal pitch is 2.5 m above the ground.
A kicker kicks a ball from a point O with a speed of $u\,\text{m s}^{-1}$ and at an angle of 30° above the horizontal.
The ball just clears the crossbar.
The horizontal component of the distance travelled by the ball until it clears the crossbar is 50 m.
Model the ball as a particle.

(iii) Find u [3]

(iv) Find the maximum height reached by the ball. [3]

THIS IS THE END OF THE QUESTION PAPER