Rewarding Learning

ADVANCED
General Certificate of Education
January 2009

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

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WEDNESDAY 21 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the two lines whose vector equations are

$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and

$\mathbf{r}_2 = \mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ [8]

2 A curve is defined by the parametric equations

$$x = 2t \qquad y = t^3 - 3t$$

Find in terms of t

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ [4]

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ [3]

3 A body is moving so that the rate of change of its distance from a fixed point A is inversely proportional to its distance x from A at any time t.
This can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k}{x}$$

where k is a constant.

Given that $x = 100$ m when $t = 0$ seconds and

that $x = 50$ m when $t = 5$ seconds,

find the total time the body takes to reach A. [10]

4 **(i)** Sketch the graph of $y = \cos x$ where $0 \leqslant x \leqslant \pi$ [2]

(ii) Hence sketch the graph of $y = \cos^{-1} x$ where $0 \leqslant y \leqslant \pi$ [3]

5 **(i)** Write $\dfrac{3x + 4}{x(x + 1)}$ in partial fractions. [6]

(ii) Hence find the exact area bounded by the curve $y = \dfrac{3x + 4}{x(x + 1)}$, the x-axis and the lines $x = 2$ and $x = 3$ [7]
[The curve does not cross the x-axis between 2 and 3]

6 The functions f and g are defined by:

$$\begin{aligned} &\mathrm{f} : x \to 2x + 5 \qquad && x \in \mathbb{R} \quad x \geqslant 0 \\ &\mathrm{g} : x \to |x| \qquad && x \in \mathbb{R} \quad x > -1 \end{aligned}$$

(i) State the range of f. [1]

(ii) Find the composite function fg, stating its domain. [4]

(iii) Find the inverse function f^{-1}, stating its domain and range. [5]

7 **(i)** Prove the identity

$$\sin 3A \equiv 3 \sin A - 4 \sin^3 A$$ [7]

(ii) Hence solve the equation

$$\sin A + \sin 3A = 0$$

where $0° \leqslant A \leqslant 360°$ [8]

8 Find

$$\int x \operatorname{cosec}^2 x \, \mathrm{d}x$$ [7]

THIS IS THE END OF THE QUESTION PAPER