General Certificate of Education
June 2005
Advanced Level Examination

# MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Pure 7

**MBP7**

Wednesday 22 June 2005 Afternoon Session

**In addition to this paper you will require:**
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP7.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** Find $\lim_{x \to 0} \left( \frac{x + \sin x}{1 - e^{-x}} \right)$. *(3 marks)*

**2** On a sketch of the complex plane, shade the region represented by the inequality

$$|z - (2 + i)| < |z + 1|$$ *(3 marks)*

**3** The cubic equation $x^3 - 5x^2 + 6x + 11 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

Determine the values of:

(a) $\alpha + \beta + \gamma$; *(1 mark)*

(b) $\alpha^2 + \beta^2 + \gamma^2$; *(3 marks)*

(c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. *(2 marks)*

**4** A line $L$ has cartesian equations $x - 7 = \frac{y + 4}{7} = \frac{z - 37}{6}$.

(a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, a vector equation for $L$. *(2 marks)*

(b) Determine the shortest distance from $P(-25, 6, 2)$ to $L$. *(5 marks)*

**5** The system of equations

$$\begin{aligned} x - \quad y + 2z &= 26 \\ 2x + \quad y + 3z &= 47 \\ 4x + 35y - 5z &= 39 \end{aligned}$$

is consistent.

(a) Show that this system of equations has no unique solution. *(2 marks)*

(b) Find the solution of this system of equations. *(5 marks)*

(c) Interpret this solution geometrically. *(1 mark)*

**6** (a) The set $S$ consists of all matrices of the form $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, where $a$ is a real number.

Show that $S$, under the operation of matrix multiplication, forms a group $G$.

*(You may assume that matrix multiplication is associative.)* *(4 marks)*

(b) Determine, giving a reason in each case, whether the group $G$ is isomorphic to:

(i) the set of real numbers under addition; *(2 marks)*

(ii) the set of real numbers under multiplication. *(2 marks)*

**7** A curve has polar equation $r = \frac{4}{9} + \ln\left(1 + \frac{\theta}{3}\right)$.

(a) (i) Write out the series expansion for $\ln\left(1 + \frac{\theta}{3}\right)$ in ascending powers of $\theta$, up to and including the term in $\theta^2$, and state the range of values of $\theta$ for which the full expansion is valid. *(3 marks)*

(ii) Use this result to write down a linear approximation for $\frac{\mathrm{d}r}{\mathrm{d}\theta}$. *(1 mark)*

(b) Given that $\theta$ is sufficiently small for terms in $\theta^2$ and higher powers of $\theta$ to be ignored, show that

$$r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 \approx \frac{1}{81}(25 + 18\theta)$$ *(3 marks)*

(c) Hence find an approximation to the length of the arc of this curve between the points where $\theta = -0.5$ and $\theta = 0$, giving your answer to 3 decimal places. *(4 marks)*

**TURN OVER FOR THE NEXT QUESTION**

**8** An ellipse has parametric form

$$x = 2\cos\theta\,, \quad y = \tfrac{1}{2}\sin\theta \quad (0 \leqslant \theta \leqslant 2\pi)$$

(a) Show that the tangent to this ellipse at the point $P(2\cos\theta\,,\ \frac{1}{2}\sin\theta)$ can be written in the form

$$x\cos\theta + 4y\sin\theta = 2 \qquad \textit{(4 marks)}$$

(b) (i) Show that this tangent meets the hyperbola with equation $x^2 - 9y^2 = 9$ when

$$(25\sin^2\theta - 9)y^2 - (16\sin\theta)y + (9\sin^2\theta - 5) = 0 \qquad \textit{(4 marks)}$$

(ii) The tangent to the ellipse is also a tangent to the hyperbola.

Find all possible values of $\sin\theta$. *(6 marks)*

**END OF QUESTIONS**