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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBP7 Pure 7

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
$-x$ ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted x marks for mis-copy
MR – *x*	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Pure 7 MBP7 June 2005

Q	Solution	Marks	Total	Comments
1	Checking nr. = dr. = 0 at $x = 0$	B1		i.e. justifying use of L'Hopital
	For evaluating $\frac{1+\cos x}{e^{-x}}$ at $x = 0$	M1		
	For answer 2	A1	3	cso
	ALTERNATIVE:			
	$\frac{x+\sin x}{1\text{-}e^{-x}} = \frac{2x - \frac{1}{6}x^3 ...}{1-\left(1-x+\frac{1}{2}x^2 ...\right)}$	(B1)		Both series correctly quoted
	$= \frac{2-\frac{1}{6}x^2 + ...}{1-\frac{1}{2}x+...} = 2$	(M1)		
		(A1)	(3)	
	Total		**3**	
2	For correctly identifying/plotting $2 + \text{i}$ and -1	B1		
	Perpendicular bisector of line segment joining the above points	B1√		ft
	Correct region (to RHS of p.b.)	B1√	3	ft. Ignore inclusion of line
	ALTERNATIVE:			
	For identifying perp. bisr. $y = 3 - 2x$	(B1)		
	For identifying correct region	(M1)		
		(A1)	(3)	
	Total		**3**	
3(a)	$\Sigma\alpha = 5$	B1	1	
(b)	$\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)$	M1		
	$\Sigma\alpha\beta = 6$	B1		
	$= 25 - 2 \times 6 = 13$	A1√	3	ft (a) and $\Sigma\alpha\beta$
(c)	$\Sigma\left(\frac{1}{\alpha}\right) = \frac{\Sigma\alpha\beta}{\alpha\beta\gamma} = -\frac{6}{11}$	M1		
		A1	2	
	ALTERNATIVE:			
	$\Sigma\alpha^2 - 5\,\Sigma\alpha + 6\,\Sigma 1 + 11\,\Sigma\left(\frac{1}{\alpha}\right) = 0$	(M1)		i.e. dividing eqn. by x, subst^{g}. α, β, γ and adding
	$\Rightarrow 13 - 25 + 18 + 11\,\Sigma\left(\frac{1}{\alpha}\right) = 0$			
	$\Rightarrow \Sigma\left(\frac{1}{\alpha}\right) = -\frac{6}{11}$	(A1√)	(2)	ft (b) – 5 × (a) + …
	Total		**6**	

MBP7 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{r} = \begin{bmatrix} 7 \\ -4 \\ 37 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}$	B1		Any suitable p.v. of pt. on line
		B1	2	Any d.v.
(b)	Use of $\overrightarrow{PR} = \begin{bmatrix} \lambda+32 \\ 7\lambda-10 \\ 6\lambda+35 \end{bmatrix}$	B1		
	$\lvert\overrightarrow{PR}\rvert^2 = (\lambda+32)^2 + (7\lambda-10)^2 + (6\lambda+35)^2$ $= 86\lambda^2 + 344\lambda + 2349$	M1		Attempt at magnitude^2
	Diff^g. this w.r.t. λ: $172\lambda + 344$	m1		
	$= 0$ when $\lambda = -2$	A1		
	$\Rightarrow PR_{\min} = \sqrt{2005}$	B1√	5	ft their $\sqrt{PR^2}$ with their λ
	ALTERNATIVE 1:			
	Use of $\overrightarrow{PR} = \begin{bmatrix} \lambda+32 \\ 7\lambda-10 \\ 6\lambda+35 \end{bmatrix}$	(B1)		
	Attempt at $\overrightarrow{PR} \bullet \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} = 0$	(M1)		
	Solving eqn. for λ: $186\lambda + 172 = 0$	(m1)		
	$\lambda = -2$	(A1)		
	$\Rightarrow PR_{\min} = \sqrt{2005}$	(B1√)	(5)	ft their $\lvert\overrightarrow{PR}\rvert$ with their λ
	ALTERNATIVE 2:			
	Sh. D. = $\dfrac{\lvert \mathbf{b} \times (\mathbf{p} - \mathbf{a}) \rvert}{\lvert \mathbf{b} \rvert}$	(M2)		Used
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 7 & 6 \\ -32 & 10 & -35 \end{vmatrix} = -305\mathbf{i} - 157\mathbf{j} + 234\mathbf{k}$	(m1) (A1)		
	Sh. D. = $\dfrac{\sqrt{172430}}{\sqrt{86}} = \sqrt{2005} \approx 44.777$	(A1√)	(5)	ft. At least 3 s.f. if only decimal answer given
	Total		**7**	

MBP7 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Evaluating $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 4 & 35 & -5 \end{vmatrix}$	M1		
	= (– 5 + 140 – 12) – (8 + 105 + 10) = 0	A1	2	
	ALTERNATIVE: $\left[\begin{array}{ccc\|c} 1 & -1 & 2 & 26 \\ 2 & 1 & 3 & 47 \\ 4 & 35 & -5 & 39 \end{array}\right]$	(M1)		E.g. $R_2^* = R_2 - 2R_1$ $R_3^* = R_3 - 4R_1$
	$\rightarrow \left[\begin{array}{ccc\|c} 1 & -1 & 2 & 26 \\ 0 & 3 & -1 & -5 \\ 0 & 39 & -13 & -65 \end{array}\right]$	(A1)	(2)	N.B. Observing that $R_3^* = 13\ R_2^*$ now gives (a)'s result from here. Etc.
(b)	$E_2^* = E_2 - 2E_1 \Rightarrow 3y - z = -5$ Setting (e.g.) $y = \lambda$ so that $z = 3\lambda + 5$ and $x = 16 - 5\lambda$	M1 A1 M1 A1 A1	5	N.B. Any method acceptable
	OR: $x = \lambda,\ y = \dfrac{16-\lambda}{5},\ z = \dfrac{73-3\lambda}{5}$ $z = \lambda,\ x = \dfrac{\lambda-5}{3},\ z = \dfrac{73-5\lambda}{3}$			OR equivalent answers
(c)	The line of intersection of three planes	B1	1	
	Total		**8**	
6(a)	$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} \Rightarrow$ Closed	M1 A1		
	The identity, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is in S	B1		
	$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} =$ is in S	B1	4	
(b)(i)	$(S, \times_M) \cong (\Re, +)$ Isomorphism $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \rightarrow a$	B1 B1	2	
(ii)	$(S, \times_M)$ not $\cong (\Re, \times)$ Any good reason	B1 B1	2	E.g. $(\Re, \times)$ not a group or **AB** does not map to ab
	Total		**8**	

MBP7 (cont)

Q	Solution	Marks	Total	Comments
7(a)	**(i)** $\ln\left(1+\frac{1}{3}\theta\right) = \frac{1}{3}\theta - \frac{1}{18}\theta^2 + \ldots$	M1 A1		
	Expansion valid for $-3 < \theta \le 3$	B1	3	
	(ii) $r \approx \frac{4}{9} + \frac{1}{3}\theta - \frac{1}{18}\theta^2 + \ldots$			
	$\Rightarrow \frac{dr}{d\theta} \approx \frac{1}{3} - \frac{1}{9}\theta$	B1√	1	ft
(b)	$r^2 + \left(\frac{dr}{d\theta}\right)^2$	M1		Attempt at both
	$\approx \left(\frac{16}{81} + 2.\frac{4}{9}.\frac{1}{3}\theta\ldots\right) + \left(\frac{1}{9} - 2.\frac{1}{3}.\frac{1}{9}\theta\ldots\right)$	A1√		ft either correct, unsimplified (relevant terms only)
	$= \frac{1}{81}(25 + 18\theta)$	A1	3	Answer given
(c)	$L = \int \frac{1}{9}(25+18\theta)^{1/2}\, d\theta$	B1		
	$= \frac{1}{243}(25+18\theta)^{3/2}$	M1		Correct power of $(25 + 18\theta)$
		A1		Correct multiple
	= 0.251 (to 3 d.p.)	A1	4	Exactly this (but allow $\frac{61}{243}$)
	Total		**11**	
8(a)	$\frac{dy}{dx} = \frac{\frac{1}{2}\cos\theta}{-2\sin\theta}$	M1 A1		
	Eqn. tgt. is $y - \frac{1}{2}\sin\theta = -\frac{\cos\theta}{-4\sin\theta}(x - 2\cos\theta)$	M1		Or $y = -\frac{\cos\theta}{-4\sin\theta}x + C$ with $(2\cos\theta, \frac{1}{2}\sin\theta)$ substd. to find C
	$4y\sin\theta - 2\sin^2\theta = -x\cos\theta + 2\cos^2\theta$			
	$\Rightarrow x\cos\theta + 4y\sin\theta = 2$	A1	4	Answer given. N.B. There is a quoteable form for the tgt. if the ellipse eqn. can be identified. This is fine.
(b)(i)	$x^2 = \frac{4 - 16y\sin\theta + 16y^2\sin^2\theta}{\cos^2\theta}$	B1		
	Use of $c^2 = 1 - s^2$ at any stage	B1		
	Subst^g. for x into $x^2 - 9y^2 = 9$	M1		
	$(25s^2 - 9)y^2 - 16s\,y + (9s^2 - 5) = 0$	A1	4	Getting given answer legitimately
(ii)	Considering the discriminant	M1		
	Equate to zero + attempt to solve	m1		
	$\Rightarrow 256s^2 = 4(9s^2 - 5)(25s^2 - 9)$	A1		Or ≡ with factors cancelled
	$\Rightarrow 0 = m(5s^4 - 6s^2 + 1)$	A1		
	$(5s^2 - 1)(s^2 - 1) = 0$	M1		ft quadratic in s^2
	$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{5}}$ and ± 1	A1	6	cao
	Total		**14**	
	TOTAL		**60**	