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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBP5 Pure 5

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

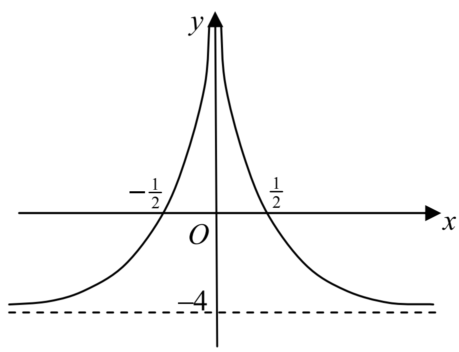
Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Pure 5 MBP5 June 2005

Q	Solution	Marks	Total	Comments
1	$h = 0.5$	B1		
	Integral = $h/2\{\ldots\}$			
	$\{\ldots\}=\left[\mathrm{f}(1)+\mathrm{f}(2.5)+2\left(\mathrm{f}(1.5)+\mathrm{f}(2)\right)\right]$	M1		Where $\mathrm{f}(x) = 2^x - 1$.
	$=\{1+(4\sqrt{2}-1) + 2[(2\sqrt{2}-1) + 3]\}$ $\{4\sqrt{2}-1=4.65685..\}$ $\{2\sqrt{2}-1=1.82842\ldots\}$	A1		All 4 terms correct.[accept 3 dp or better for each term or 15.31(37…)seen or 3.82(8…) seen if index or surd form not given]
	Integral to 3sf = 3.83	A1	4	cao Must be 3.83
	Total		**4**	
2(a)	$(1+8x)^{\frac{1}{2}} = 1 + kx + \ldots..$	M1		Valid start to binomial expn.
	$\left(\left(\frac{1}{2}\right)(8x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(8x)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(8x)^3+\ldots\right)$			
	$=1+4x$	A1		
	$-8x^2$	A1		
	$+32x^3$ (.....)	A1	4	Accept $a = 4$, $b = -8$, $c = 32$ **sc** if 0/3 give B1 for at least two unsimplified terms correct in {} above
(b)	Valid for $-\frac{1}{8} < x < \frac{1}{8}$	B1	1	oe Accept $\lvert x \rvert \le \frac{1}{8}$ oe
	Total		**5**	
3(a)	Asymptotes: $x = 0$; $y = -4$	B1 B1	2	If no contradiction, accept **equations** of asymptotes shown on the graph
(b)		B2		Correct sketch [B1 if **either** (i) one correct branch **or** (ii) correct 2-branch shape translated **or** (iii) 2-branch curve with intended symmetry about y-axis.
		B1	3	Only pts of intersection with x-axis at –0.5 and 0.5
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 - 2x^{-3} = -2$ at (1, –3)	M1		Attempts to find y' at (1, –3) having got at least one 'term' correct in $y'(x)$
	Gradient of normal $= \frac{1}{2}$	m1		Uses $m \times m' = -1$, numerical m's. PI
	Eqn of normal $y+3=\frac{1}{2}(x-1)$	A1	3	Accept in any correct form provided cso
	Total		**8**	

MBP5 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$R\cos\alpha = 1$ or $R\sin\alpha = 1$ or $\tan\alpha = 1$	M1		Accept seen; condone negative signs PI by correct value of α.
	$R^2 = 1^2 + 1^2$	M1		Altn Use two of results in line1.
	$\Rightarrow\ R = \sqrt{2}\,,\ \ \alpha = \dfrac{\pi}{4}$	A1	3	Need both:Accept $R = 1.41$, $\alpha = 0.78(5)$; condone $\alpha = 45°$
(b)	$\sin(x + \alpha) = \dfrac{1}{R\sqrt{2}}$ {=0.5}	M1		Using (a) to reach $\sin(x+\alpha) = k$ PI
	$x + \alpha = 2\pi n + \sin^{-1}$ (*), also $x + \alpha = 2\pi n + [\pi - \sin^{-1}$ (*)], (*=cand's $1/(R\sqrt{2})$.)	m1		Accept degrees, rads., mix but need both sets of gen. solns….(watch out for valid equivalents)
	$x + \alpha = 2\pi n + \dfrac{\pi}{6}$ $x + \alpha = 2\pi n + \pi - \dfrac{\pi}{6}$	A1		oe condone degrees or mix but need both sets.
	$x = 2\pi n - \dfrac{\pi}{12}$; $2\pi n + \dfrac{7\pi}{12}$ [$x=2\pi n -${0.261 to 0.262 inclusive} $x=2\pi n$ + {1.83 to 1.84 inclusive}]	A1	4	Any equivalent general forms for x in radians. **sc** if m0 then award B1 for either one general soln. or 2 particular solns. covering both branches condone degrees or mix. <u>In (c) do **NOT** penalise wrong values for R and α.</u>
(c)	$\int ...... = R\int x\sin(x+\alpha)\,dx$	M1		Use of part (a)
	$\int x\sin(x+\alpha)\,dx =$			
	$-x\cos(x+\alpha) + \int\cos(x+\alpha)\,dx$	m1 A1		Condone sign errors or clear miscopy only Correct integration
	$\int ... = R\{-x\cos(x+\alpha) + \sin(x+\alpha)\} + c$	A1√	4	ft sign errors in previous result. Condone absence of $+c$
	<u>ALT</u> Attempts to integrate <u>both</u> $x\sin x$ and $x\cos x$ by parts	(M1)		
	$\int x\sin x = -x\cos x + \int\cos x$ $\int x\cos x = x\sin x - \int\sin x$	(m1) (A1)		Condone sign errors only. cao both
	$... = -x\cos x + x\sin x + \sin x + \cos x + c$	(A1√)		ft sign errors in previous result and condone abs. $+c$
	Total		**11**	

MBP5 (cont)

Q	Solution	Marks	Total	Comments
5(a)	At (0, –1), $\frac{dy}{dx} = \frac{0}{-1} = 0$	B1	1	
	(so P is a stationary point)			(so…..) not required
(b)(i)	$\frac{d^2y}{dx^2} = \frac{y(1) - x\frac{dy}{dx}}{y^2}$	M1		Clear use of quotient rule [or relevant product rule]
	$\frac{d^2y}{dx^2} = \frac{y(1) - x\left(\frac{x}{y}\right)}{y^2}$	m1		Subst. of $\frac{dy}{dx} = \frac{x}{y}$ oe
	$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}$	A1	3	**ag** cso
(ii)	At (0, –1), $\frac{d^2y}{dx^2} = \frac{1-0}{-1} = -1 < 0$			
	so P is a maximum point	B1	1	cso
(c)	$y\,dy = x\,dx$	M1		Clear attempt to separate variables
	$\frac{y^2}{2} = \frac{x^2}{2} + c$	A1		$\frac{y^2}{2} = \frac{x^2}{2}$
	$\frac{1}{2} = \frac{0}{2} + c$	m1		Use of boundary conditions
	$y^2 = x^2 + 1$	A1	4	
	Total		**9**	
6(a)(i)	$\begin{pmatrix}1\\2\\4\end{pmatrix} . \begin{pmatrix}2\\2\\1\end{pmatrix} = 2 + 4 + 4 = 10$	B1	1	
(ii)	Magnitude of direction vectors are $\sqrt{21}$ and $\sqrt{9}$	B1		Award for one correct.
	$10 = \sqrt{21} \times \sqrt{9} \cos\theta$	M1		Use of dot product (ft on earlier values)
	$\cos\theta = \frac{10}{\sqrt{189}}$ {=0.72739….}			
	$\Rightarrow \theta = 43.3°$ {= 43° to nearest degree}	A1	3	**ag** Accept without seeing 3sf if clear
(b)	$(2+s) + (-1+2s) + (-2+4s) = 20$	M1		
	$\Rightarrow s = 3$	A1		
	$\Rightarrow Q$ has position vector 5**i**+5**j**+10**k**	A1√	3	ft a slip in finding s **sc** (cand uses l_2): Mark as max M1A0A1 {8.6**i** + 6.6**j** + 4.8**k**} Condone (5,5,10) notation.
	Total		**7**	

MBP5 (cont)

Q	Solution	Marks	Total	Comments
7(a)	When $t = \frac{\pi}{2}$, $x = 4\sin\frac{\pi}{2} = 4$ and $y = \cos\frac{\pi}{2} = 0$ ie (4,0)	B1	1	**ag** Accept any valid method
(b)	$\frac{dx}{dt} = 4\cos t$, $\frac{dy}{dt} = -\sin t$	M1		Attempts both and at least one correct (possibly implied)
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin t}{4\cos t} = -\frac{1}{4}\tan t$	A1	2	**ag** obtained convincingly
(c)(i)	At P, $x = 4\sin\frac{\pi}{4}$, $y = \cos\frac{\pi}{4}$	M1		oe Accept $x = 2.82$ to 2.83 inclusive Accept $y =$ awrt 0.71
	grad of tang. $= -\frac{1}{4}\tan\frac{\pi}{4} = -\frac{1}{4}$	B1		
	Eq tang, $y - \cos\frac{\pi}{4} = -\frac{1}{4}\left(x - 4\sin\frac{\pi}{4}\right)$	M1		oe
	$\Rightarrow y - \frac{1}{\sqrt{2}} = -\frac{1}{4}x + \frac{1}{\sqrt{2}}$			
	$\Rightarrow y = -\frac{1}{4}x + \sqrt{2}$	A1	4	**ag** cso obtained convincingly
(ii)	When $y = 0$, $x = 4\sqrt{2}$; When $x = 0$, $y = \sqrt{2}$	M1		Attempts to find pts where tangent intersects axes
	Area of triangle = 4	A1		Must be justified
	Area shaded = area of Δ– area 'under curve'	M1		
	area 'under curve' $= \int_0^{\frac{\pi}{2}} y\frac{dx}{dt}dt$	M1		Need attempt to write integrand in terms of t. Condone wrong/missing limits
	$= \int \cos t(4\cos t)dt$	A1		Ignore limits
	Area shaded $= 4 - \int_0^{\frac{\pi}{2}} 4\cos^2 t\ dt =$ pr. ans	A1	6	**ag** cso be convinced; correct limits should have appeared before the printed answer stage
(iii)	Area shaded $= 4 - 2\int_0^{\frac{\pi}{2}} (1 + \cos 2t)dt$	M1		Use of $2\cos^2 t = 1 + \cos 2t$ (condone sign errors)
	…. $= 4 - 2\left[t + \frac{1}{2}\sin 2t\right]_0^{\frac{\pi}{2}}$	A1		for […..]
	… $= 4 - \pi$	A1	3	cso
	Total		**16**	
	TOTAL		**60**	