

General Certificate of Education
June 2005
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 3**

MBP3

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) A curve has equation $y = \frac{4x - 3}{2 - x}$.

(i) Find the coordinates of the points where the curve crosses the coordinate axes. (2 marks)

(ii) State the equations of its asymptotes. (2 marks)

(iii) Sketch the curve. (2 marks)

(b) Hence, or otherwise, solve the inequality

$$\frac{4x - 3}{2 - x} < 1 \quad (3 \text{ marks})$$

2 (a) The roots of the quadratic equation $x^2 - 3x + 5 = 0$ are α and β .

(i) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. (2 marks)

(ii) Without solving the quadratic equation, find the value of $\alpha^2 + \beta^2$.

Hence explain why α and β cannot both be real. (3 marks)

(iii) Show that $\alpha^3 + \beta^3 = -18$. (2 marks)

(b) Determine a quadratic equation with integer coefficients which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. (4 marks)

3 The matrix \mathbf{M} is $\begin{bmatrix} k & -7 \\ -2 & 5 \end{bmatrix}$. The transformation \mathbf{T} is given by $\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$.

(a) Find the determinant of \mathbf{M} , giving your answer in terms of k . (1 mark)

(b) A quadrilateral is transformed by \mathbf{T} into a quadrilateral with the same area as the original. Find the two values of k for which this occurs. (3 marks)

(c) When $k = 4$:

(i) find the inverse matrix \mathbf{M}^{-1} ; (2 marks)

(ii) find the coordinates of the point which is mapped onto $(1, 7)$ under \mathbf{T} . (3 marks)

- 4 (a) Find the complex roots of the quadratic equation

$$x^2 - 4x + 13 = 0 \quad (3 \text{ marks})$$

- (b) Given that $(p + 3i)^2 = q + 12i$, find the value of each of the real numbers p and q .
(3 marks)

- 5 Prove by mathematical induction that, for all positive integers n ,

$$\sum_{r=1}^n \frac{r}{2^r} = 2 - \frac{(n+2)}{2^n} \quad (5 \text{ marks})$$

- 6 (a) A curve has equation

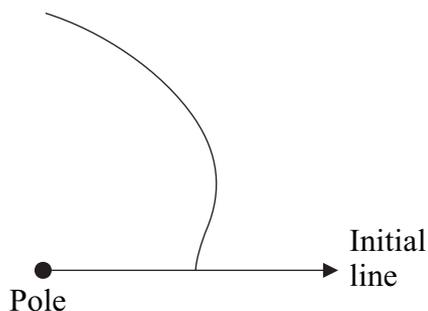
$$r = 3 - 2 \cos \theta$$

for $-\pi < \theta \leq \pi$, where $[r, \theta]$ are polar coordinates.

- (i) State the maximum and minimum values of r .

For each of these values of r , give a corresponding value of θ . (4 marks)

- (ii) The curve is shown below for $0 \leq \theta \leq \frac{\pi}{2}$. Sketch the curve for $-\pi < \theta \leq \pi$.



(2 marks)

- (b) (i) Find the values of $\cos \theta$ for which

$$8 \cos^2 \theta = 3 - 2 \cos \theta \quad (2 \text{ marks})$$

- (ii) Find the polar coordinates of the points of intersection of the curve with equation $r = 3 - 2 \cos \theta$ and the curve with equation $r = 8 \cos^2 \theta$. (4 marks)

Turn over ►

7 The value of a car, £ V , can be modelled by the equation

$$V = 15\,000 - 5\,000\sqrt{t}, \quad 0 \leq t \leq 8$$

where t is the time, in years, measured from when the car is new.

(a) Use this model to find the value of the car when:

(i) it is new; (1 mark)

(ii) it is 4 years old. (1 mark)

(b) (i) Find the value of $\frac{dV}{dt}$ when $t = 4$. (3 marks)

(ii) Interpret your answer to part (b)(i) in the context of this model. (2 marks)

(c) A second model relating V and t is proposed.

It is given by $V = a \times b^{-t}$, where a and b are constants.

(i) Express $\log_{10} V$ in terms of $\log_{10} a$, $\log_{10} b$ and t . (2 marks)

(ii) Given that $V = 11\,500$ when $t = 1$ and $V = 5\,000$ when $t = 4$, find the value of a and the value of b , giving your answers to three significant figures. (4 marks)

8 The binary operation \otimes is defined by $p \otimes q = p + q + pq$, where p and q are real numbers.

(a) Show that the identity element is 0. (1 mark)

(b) Find the inverse of 3. (3 marks)

(c) (i) Express $(p \otimes q) \otimes r$ in a form not involving \otimes . (2 marks)

(ii) Hence determine whether \otimes is associative. (2 marks)

9 (a) Sketch the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$, marking clearly the values of any intercepts with the coordinate axes. (3 marks)

(b) Describe the geometrical transformation that maps the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$ onto the graph of $\frac{(x+1)^2}{4} - \frac{y^2}{36} = 1$. (4 marks)

END OF QUESTIONS