General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP2**
(SPECIFICATION B)
Unit Pure 2

Thursday 9 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A geometric series begins

$$8 + 4 + 2 + 1 + \dots$$

(a) (i) State the value of the common ratio of the series. *(1 mark)*

(ii) Give a reason why the series is convergent. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the 24th term of the series, giving your answer in the form 2^k, where k is a negative integer. *(3 marks)*

2 The function f is defined for all real values of x by $\mathrm{f}(x) = (x+1)(x-1)(x-4)$.

(a) (i) Write down the **three** values of x for which $\mathrm{f}(x) = 0$. *(2 marks)*

(ii) Sketch the curve with equation $y = \mathrm{f}(x)$. Indicate the coordinates of the four points where the curve crosses the axes.

(You are not required to calculate the coordinates of the stationary points.) *(2 marks)*

(iii) Hence solve the inequality $\mathrm{f}(x) > 0$. *(2 marks)*

(b) (i) Express $\mathrm{f}(x)$ in the form $x^3 + px^2 + qx + 4$, where p and q are integers to be found. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{(x+1)(x-1)(x-4)}{x}\,\mathrm{d}x$. *(5 marks)*

(iii) Hence show that $\displaystyle\int_1^2 \frac{(x+1)(x-1)(x-4)}{x}\,\mathrm{d}x = 4\ln 2 - \frac{14}{3}$. *(2 marks)*

3 A polynomial is given by $\mathrm{p}(x) = 2x^3 - 3x^2 - 3x + 2$.

(a) Use the factor theorem to show that $2x - 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(b) Find the value of $\mathrm{p}(-1)$. *(1 mark)*

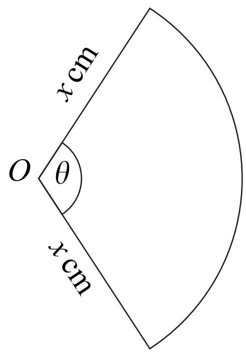
(c) Express $\mathrm{p}(x)$ as a product of three linear factors. *(3 marks)*

(d) Hence find the three values of y that satisfy the equation

$$2(\ln y)^3 - 3(\ln y)^2 - 3\ln y + 2 = 0$$

giving each answer in the form e^k, where k is a constant. *(4 marks)*

4 The diagram shows a sector of a circle with centre O and radius x cm. The angle of the sector is θ radians, where $0 < \theta < \pi$.



The area of the sector is $16\,\text{cm}^2$. The perimeter of the sector is P cm.

(a) Find θ in terms of x. *(2 marks)*

(b) Hence show that $P = 2x + \dfrac{32}{x}$. *(3 marks)*

(c) (i) Find $\dfrac{\mathrm{d}P}{\mathrm{d}x}$. *(2 marks)*

(ii) Find the value of x for which P has a stationary value. *(2 marks)*

(d) (i) Find $\dfrac{\mathrm{d}^2P}{\mathrm{d}x^2}$. *(1 mark)*

(ii) Hence determine whether the stationary value of P is a maximum or a minimum. *(2 marks)*

Turn over ▶

5 (a) Write down, in surd form, the value of:

(i) $\cos\dfrac{\pi}{4}$; *(1 mark)*

(ii) $\cos\dfrac{5\pi}{6}$. *(1 mark)*

(b) (i) Write down the four **exact** values of $\cos\theta$ that satisfy the equation

$$(4\cos^2\theta - 3)(2\cos^2\theta - 1) = 0$$ *(2 marks)*

(ii) Hence find the four values of θ in the interval $0 < \theta < \pi$ that satisfy the equation

$$(4\cos^2\theta - 3)(2\cos^2\theta - 1) = 0$$

Give your answers in terms of π, in a simplified exact form. *(4 marks)*

6 It is given that $y = 9\mathrm{e}^{2x}$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(b) (i) Show that $x = k\ln y - \ln 3$, where k is a constant to be found. *(4 marks)*

(ii) Use this expression for x to find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. *(1 mark)*

(c) Use your answers to parts (a) and (b) to verify that $\dfrac{\mathrm{d}y}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}y} = 1$. *(1 mark)*

END OF QUESTIONS