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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBP2 Pure 2

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
$-x$ ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

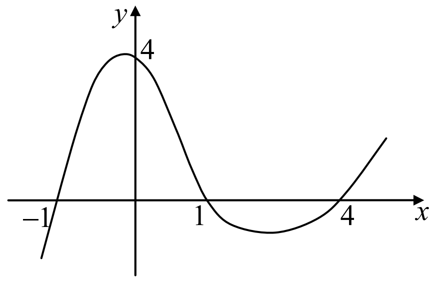
Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Pure 2 MBP2 June 2005

Q	Solution	Marks	Total	Comments
1(a)(i)	0.5	B1	1	
(ii)	When $r = 0.5$, $-1 < r < 1$ so series is convergent	E1	1	oe
(b)	$S_\infty = \dfrac{a}{1-r}$	M1		
	…. = 16	A1√	2	ft only of value of r such that $\lvert r \rvert < 1$
(c)	24th term = ar^{24-1}	M1		Condone ar^{24}
	……….. $= 8\times\left(\dfrac{1}{2}\right)^{23}$	A1		oe Accept 9.54×10^{-7} or better
	……….. $= 2^{-20}$	A1	3	(accept $k = -20$ if clear)
	Total		**7**	
2(a)(i)	− 1, 1 and 4	B2,1,0	2	
(ii)		B1		<u>Cubic Shape</u>: one minimum and one max to left of min. (Condone max on or to the right of the y-axis)
		B1√	2	Cubic cutting x-axis at three ft values from (i) and cutting y-axis at 4.
(iii)	$x > 4$	B1√		If incorrect, ft on cubic graph with three points of intersection with x-axis.
	$-1 < x < 1$	B1√	2	Deduct maximum of 1 mark for use of non-strict inequalities
(b)(i)	$\mathrm{f}(x) = (x^2-1)(x-4) = x^3 - 4x^2 - x + 4$	M1		Attempts to multiply the remaining 'bracket' by product of any two brackets
		A1	2	Accept $p = -4$, $q = -1$
(ii)	$\dfrac{\mathrm{f}(x)}{x} = \dfrac{x^3 - 4x^2 - x + 4}{x}$			
	.... $= x^2 - 4x - 1 + \dfrac{4}{x}$	M1A1√		M1 (one term ft correct)
	$\displaystyle\int \frac{\mathrm{f}(x)}{x}\,\mathrm{d}x = \frac{x^3}{3} - \frac{4x^2}{2} - x + 4\ln x + c$	A1		$\dfrac{x^3}{3}$
		A1		$4\ln x$
		A1√	5	$\dfrac{px^2}{2} + qx$ (Condone absence of '+c')
(iii)	$\displaystyle\int_1^2 ..... = \left(\frac{8}{3} - 8 - 2 + 4\ln 2\right) - \left(\frac{1}{3} - 2 - 1\right)$	M1		F(2) − F(1)
	….. $= \dfrac{7}{3} - 7 + 4\ln 2 = 4\ln 2 - \dfrac{14}{3}$	A1	2	cso ag
	Total		**15**	

MBP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\text{p}\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 2$	M1		Use of $\text{p}\left(\frac{1}{2}\right)$
	$= 0 \Rightarrow (2x - 1)$ is a factor of p(x)	A1	2	ag Must have the conclusion.
(b)	$\text{p}(-1) = -2-3+3+2 = 0$	B1	1	
(c)	(x + 1) is a factor of p(x)	B1		Award at any stage
	$\text{p}(x) \equiv (x + 1)(2x - 1)[x \ldots. - 2]$	M1		Valid attempt at 3rd factor/complete method. [coeff of x^3 correct or const correct or use of p(2)]
	$\text{p}(x) \equiv (x + 1)(2x - 1)(x - 2)$	A1	3	
(d)	$x \to \ln y \Rightarrow$			
	$(\ln y + 1)(2\ln y - 1)(\ln y - 2) = 0$	M1		Using $x = \ln y$
	$\Rightarrow \ln y = -1;\ \ln y = \frac{1}{2};\ \ln y = 2$			
	$\Rightarrow y = \text{e}^{-1};\ y = \text{e}^{0.5};\ y = \text{e}^2.$	m1 A2,1√	4	for $\ln y = k$ to $y = \text{e}^k$. ft on cnd's 3rd factor (x–a) only if $a \neq 0$, $a \neq -1$, $a \neq 0.5$ (A1ft for one of the three solutions). Condone $\frac{1}{\text{e}}$ or √e forms **NMS** Mark as B4,0 provided answer to part (c) is correct.
	Total		**10**	

MBP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		For $\frac{1}{2}r^2\theta$ oe
	$16 = \frac{1}{2}x^2\theta \Rightarrow \theta = \frac{32}{x^2}$	A1	2	
(b)	{Arc=} $r\theta$	M1		Accept seen in any part
	Perimeter $P = 2x$ + Arc	M1		
	$P = 2x + x\theta = 2x + \frac{32}{x}$	A1	3	cso ag
(c)(i)	$\frac{dP}{dx} = 2 - 32x^{-2}$	B1 B1	2	B1 for each term
(ii)	$\frac{dP}{dx} = 0 \Rightarrow 2 = 32x^{-2}$	M1		Put $P'(x)$=0 and then to stage $ax^n=k$
	$\Rightarrow x^2 = 16 \Rightarrow x = 4$	A1	2	Condone ± 4
(d)(i)	$\frac{d^2P}{dx^2} = 64x^{-3}$	B1√	1	Only ft on a numerical/sign slip
(ii)	When $x = 4$, $\frac{d^2P}{dx^2} > 0$	M1		Considers sign of $\frac{d^2P}{dx^2}$ or value of $\frac{d^2P}{dx^2}$ at the stationary point
	$\Rightarrow$ Stationary value is a minimum	A1√	2	ft on c and 's $P''(x)$, no further errors seen
	Total		**12**	
5(a)(i)	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	B1	1	Accept any correct **exact** form
(ii)	$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$	B1	1	Accept any correct **exact** form
(b)(i)	$\cos\theta = \pm\frac{\sqrt{3}}{2}; \quad \pm\frac{1}{\sqrt{2}}$	B2,1	2	Any correct **exact** form (B1 for two of the four correct)
(ii)	$\theta = \frac{\pi}{6}; \frac{\pi}{4}$, from correct positive $\cos\theta$	B1 B1		Deduct maximum of 1 mark for answers in degrees or decimals (3sf or better).
	$\theta = \frac{5\pi}{6}; \frac{3\pi}{4}$ from correct negative $\cos\theta$	B1 B1	4	Ignore answers outside the given interval.
	Total		**8**	

MBP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{dy}{dx} = 18\,e^{2x}$	M1 A1	 2	For $k\,e^{2x}$, ($k \neq 9$, -9 or 0) cao
(b)(i)	$\ln y = \ln(9\,e^{2x})$ $\ldots\ldots = \ln 9 + \ln(\,e^{2x})$ $\ln y = \ln 9 + 2x$ $x = \frac{1}{2}\ln y - \frac{1}{2}\ln 9$ $x = \frac{1}{2}\ln y - \ln 3$ (k=0.5)	M1 m1 A1 A1	 4	oe Taking ln's of both sides oe Law of logs used correctly cso
(ii)	$\frac{dx}{dy} = \frac{1}{2y}$	B1√	1	ft on k : Accept $\frac{dx}{dy} = \frac{k}{y}$
(c)	$\frac{dy}{dx} \times \frac{dx}{dy} = 18\,e^{2x} \times \frac{1}{2(9\,e^{2x})} = 1$	B1	1	ag Must be convinced
	Total		**8**	
	TOTAL		**60**	