General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 6

MBM6

Monday 20 June 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM6.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The polar coordinates of a particle at time t are

$$r = 8 + t^2 \quad \text{and} \quad \theta = e^{0.1t}$$

When $t = 2$, find:

(a) the radial and transverse components of the velocity of the particle; *(4 marks)*

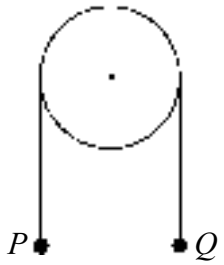
(b) the radial and transverse components of the acceleration of the particle. *(5 marks)*

2 A uniform **hollow** sphere has radius r and mass m. Initially the sphere is at rest. It then rolls without slipping down a rough plane inclined at an angle α to the horizontal.

Find the acceleration of the centre of the sphere. *(7 marks)*

3 A uniform circular disc of radius a can rotate freely in a vertical plane about a fixed horizontal axis through its centre and perpendicular to its plane. The moment of inertia of the disc about this axis is $8ma^2$.

A light inextensible string passes over the rough rim of the disc, and two particles P and Q, of masses $2m$ and $5m$ respectively, are attached to its ends.



Initially the system is held at rest with the particles hanging freely in equilibrium. The system is then released.

In the subsequent motion, the particle P remains below the disc and no slipping occurs between the string and the disc.

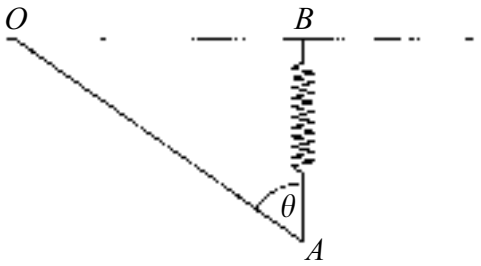
After time t, the disc has turned through an angle θ.

(a) By conservation of energy, or otherwise, show that

$$a\dot{\theta}^2 = \frac{2}{5} g\theta$$ *(6 marks)*

(b) Show that, while the disc is turning, the tensions in the two parts of the string are in the ratio 5:3. *(7 marks)*

4 A uniform rod, OA, of length $2l$ and mass m, is smoothly hinged at O. A light spring, of natural length l and modulus of elasticity $4mg$, has one end attached to A. The other end of the spring is attached to a light bead, B, which can move along a smooth horizontal wire which passes through O. The angle between the rod and the vertical is θ, as shown in the diagram.



Assume that AB is always vertical.

(a) Show that V, the total potential energy of the system, is given by

$$V = 2mgl(2\cos\theta - 1)^2 - mgl\cos\theta$$

when the potential energy is taken as zero at the level OB. *(5 marks)*

(b) Find the two values of θ, in the range $0 \leqslant \theta \leqslant \frac{\pi}{2}$, for which the rod is in equilibrium. *(4 marks)*

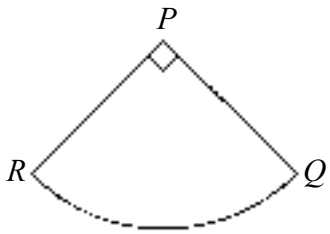
(c) Determine, at each of these positions, whether the rod is in stable or unstable equilibrium. *(5 marks)*

TURN OVER FOR THE NEXT QUESTION

5 (a) Show by integration that the moment of inertia of a uniform rod, of mass m and length $4a$, about an axis through one end of the rod and perpendicular to the rod is $\frac{16}{3}ma^2$.

(5 marks)

(b) The diagram shows the initial position of a model of a Phoenix ride at a theme park.



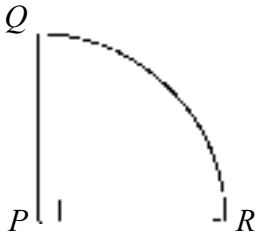
The model consists of two uniform rods, PQ and PR, and a seat in the form of a circular arc, centre P, so that angle QPR is 90°. The points P, Q and R lie in a vertical plane.

Each rod has mass m and length $4a$ and the seat has mass $5m$. The rods and the seat are rigidly fixed together and the model is free to rotate about a horizontal axis through P. The axis is perpendicular to the plane of PQR.

(i) Show that the moment of inertia of the model about this axis is $\frac{272}{3}ma^2$.

(4 marks)

(ii) The model is rotated until Q is vertically above P, as shown in the diagram below.



The model is now released from rest.

Find the greatest angular speed of the model during its subsequent motion.

You may use the fact that the centre of mass of the arc RQ is $\frac{8\sqrt{2}}{\pi}a$ from P.

(8 marks)

END OF QUESTIONS