General Certificate of Education
June 2005
Advanced Level Examination

**MATHEMATICS AND STATISTICS** **MBM5**
**(SPECIFICATION B)**
**Unit Mechanics 5**

Friday 24 June 2005 Morning Session

**In addition to this paper you will require:**
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM5.
- Answer **all** questions.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** Find the work done by a force $2\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$ acting upon a particle which moves from point $A$, with coordinates $(4, 3, -2)$, to point $B$, with coordinates $(3, 2, 6)$. *(4 marks)*

**2** A single force $(3t^2 + 7e^{-t})$ N acts at time $t$ on a particle which moves in a straight line. The force acts for 5 seconds from $t = 0$ until $t = 5$.

(a) Find the impulse on the particle in this time. *(3 marks)*

(b) The particle has mass 4 kg. When $t = 0$, the particle has velocity $6\,\text{m s}^{-1}$.

Find the velocity of the particle at the end of the 5 second period. *(2 marks)*

**3** The forces $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ act in the $x$, $y$, $z$ plane at the points with coordinates $(2, 5, 3)$, $(0, 11, 2)$ and $(5, -4, 7)$ respectively.

(a) (i) Find $\mathbf{F}$, the resultant of this system of forces. *(2 marks)*

(ii) Show that the magnitude of $\mathbf{F}$ is 13. *(2 marks)*

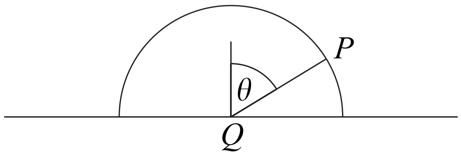
(b) The resultant of the two forces $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -3 \\ 5 \end{bmatrix}$ acts through the point $(3, 8, 7)$.

(i) Find the moment of the force $\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ about the point $(3, 8, 7)$. *(4 marks)*

(ii) Write down the moment of $\mathbf{F}$ about the point $(3, 8, 7)$. Explain your answer. *(2 marks)*

**4** A smooth hemisphere of radius $a$ and centre $Q$ lies with its plane face fixed to a horizontal surface. A particle of mass $m$ can move freely on the surface of the hemisphere.

Initially, the particle is at rest at the highest point of the hemisphere. It is set in motion along the surface of the hemisphere. The line $QP$ makes an angle of $\theta$ with the upward vertical.

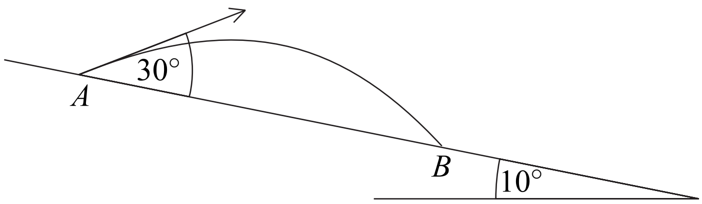



(a) Show that, when the particle is at $P$, its speed is

$$\sqrt{2ga(1-\cos\theta)}$$ *(3 marks)*

(b) Find the value of $\cos\theta$ if the particle leaves the surface of the hemisphere when it reaches $P$. *(4 marks)*

**5** Alex hits a golf ball which is lying at the point $A$ on a slope inclined at an angle of $10°$ below the horizontal. The initial velocity of the ball is $25\,\text{m s}^{-1}$ at an angle of $30°$ to the slope.




The ball first hits the slope at $B$.

Find the distance $AB$. *(8 marks)*

**6** A car is travelling around a corner on a horizontal road. The path of the car is an arc of a circle of radius 600 m.

Initially, the speed of the car is $30\,\text{m s}^{-1}$. The brakes are applied so that the speed decreases at a constant rate of $2\,\text{m s}^{-2}$.

Find the magnitude of the resultant acceleration of the car immediately after the brakes have been applied. *(5 marks)*

**Turn over ▶**

**7** A hailstone falls vertically under gravity through still air. As it falls, water vapour from the surrounding still air condenses on the hailstone causing its mass to increase. Model the hailstone as a uniform sphere. At time $t$, the hailstone has mass $m$ and radius $r$.

(a) Given that $\frac{\mathrm{d}r}{\mathrm{d}t} = \lambda r$, where $\lambda$ is a positive constant, show that $\frac{\mathrm{d}m}{\mathrm{d}t} = 3\lambda m$. *(4 marks)*

(b) Assume that the only external force acting on the hailstone is gravity.

At time $t$, the speed of the hailstone is $v$.

Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = g - 3\lambda v$. *(4 marks)*

**8** A particle $P$, of mass $m$, is suspended from a fixed point $O$ by a light elastic string of natural length $2a$ and modulus $4mn^2a$, where $n$ is a positive constant. The particle is released from rest at a point $C$, where $C$ is vertically below $O$ and $OC = 2a$.

When the particle is moving with velocity $v$, it experiences air resistance of magnitude $2mnv$.

(a) The displacement of $P$ below $C$ at time $t$ is $x$.

Show that $x$ satisfies the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 2n^2x = g$$ *(4 marks)*

(b) Find $x$ in terms of $n$, $g$ and $t$. *(9 marks)*

**END OF QUESTIONS**