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General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBM5 Mechanics 5

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Mechanics 5 MBM5 June 2005

Q	Solution	Marks	Total	Comments
1	Displacement is $\mathbf{b} - \mathbf{a} = \begin{pmatrix}3\\2\\6\end{pmatrix} - \begin{pmatrix}4\\3\\-2\end{pmatrix}$	M1		
	$= \begin{pmatrix}-1\\-1\\8\end{pmatrix}$	A1		
	Work done = **F.s** $= \begin{pmatrix}-1\\-1\\8\end{pmatrix} \cdot \begin{pmatrix}2\\-6\\5\end{pmatrix}$	M1		
	$= -2 + 6 + 40$			
	$= 44$	A1	4	
	Total		**4**	
2(a)	Using Impulse = $\int F \, \mathrm{d}t$			
	$= \int_0^5 (3t^2 + 7\mathrm{e}^{-t}) \, \mathrm{d}t$	M1		
	$= \left[t^3 - 7\mathrm{e}^{-t}\right]_0^5$	A1		
	$= 125 - 7\mathrm{e}^{-5} + 7$ $= 132 - 7\mathrm{e}^{-5}$	A1	3	
(b)	Impulse = change in momentum; $132 - 7\mathrm{e}^{-5} = 4u - 4.6$ $= 4u - 24$	M1		
	$u = 39 - \frac{7}{4}\mathrm{e}^{-5}$	A1	2	Accept 38.988
	Total		**5**	

MBM5 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\mathbf{F} = \begin{pmatrix}1\\3\\4\end{pmatrix}+\begin{pmatrix}3\\-3\\5\end{pmatrix}+\begin{pmatrix}-1\\4\\3\end{pmatrix}$	M1		
	$=\begin{pmatrix}3\\4\\12\end{pmatrix}$	A1	2	
(ii)	Magnitude = $\sqrt{3^2+4^2+12^2}$	M1		
	= 13	A1	2	
(b)(i)	Displacement of force from (3, 8, 7) is $\begin{pmatrix}5\\-4\\7\end{pmatrix}-\begin{pmatrix}3\\8\\7\end{pmatrix}$			
	$=\begin{pmatrix}2\\-12\\0\end{pmatrix}$	B1		
	Moment is $\mathbf{r}\times\mathbf{F}$			
	$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & -12 & 0\\-1 & 4 & 3\end{vmatrix}$	M1 A1		M1 if one row [and **i, j, k**] correct
	= (–-36**i** – 6**j** – 4**k**)	A1	4	**sc** 3 for 36**i** + 6**j** + 4**k**
(ii)	Moment of **F** is –-36**i** – 6**j** – 4**k**	B1√		
	Moment of the other two forces is zero since they pass through (3, 8, 7)	E1	2	
	Total		**10**	

MBM5 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Using conservation of energy			
	$\frac{1}{2}mv^2 = mga(1 - \cos\theta)$	M1 A1		
	$v^2 = 2ga(1 - \cos\theta)$			
	$v = \sqrt{2ga(1-\cos\theta}$	A1	3	
(b)	Particle leaves the surface when			
	$mg\cos\theta = \dfrac{mv^2}{a}$	M1 A1		
	$mg\cos\theta = \dfrac{m}{a} \times 2ga(1 - \cos\theta)$	M1		
	$\cos\theta = 2 - 2\cos\theta$			
	$\cos\theta = \dfrac{2}{3}$	A1	4	
	Total		**7**	
5	Distance perpendicular to slope:			
	$S = 25\sin 30\, t - \dfrac{1}{2} g \cos 10\, t^2$	M1 A1		
	Strikes plane again when $s = 0$, $t = \dfrac{50\sin 30}{g\cos 10}$	M1 A1		
	[$t = 0$ not required]			
	Distance down slope:			
	$s = 25\cos 30\, t + \dfrac{1}{2} g \sin 10\, t^2$	M1 A1		
	$= 25\cos 30 \dfrac{50\sin 30}{g\cos 10}$ $+\dfrac{1}{2} g \sin 10 \left(\dfrac{50\sin 30}{g\cos 10}\right)^2$	M1		
	$= 61.7926\ldots$ m $= 61.8$ m	A1	8	
	Total		**8**	

MBM5 (cont)

Q	Solution	Marks	Total	Comments
6	Radial component of acceleration is $\frac{v^2}{r} = \frac{900}{600}$	M1		
	$= 1.5$	A1		
	Transverse component of acceleration is $\frac{\mathrm{d}v}{\mathrm{d}t} = 2$	B1		
	Acceleration is 2.5 ms^{-2}	M1 A1	5	
	Total		**5**	
7(a)	$m = \frac{4}{3}\pi r^3 \rho$	M1		
	$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho$	A1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}$	M1		
	$= 4\pi r^2 \rho \times \lambda r$ $= 3\lambda m$	A1	4	No density used M1 A1 only
(b)	Initial $m \to v \qquad \delta m \to 0$ Final $m + \delta m$ $\to v + \delta v$ Using $F \times t$ = change in momentum $mv + mg\delta t = (m + \delta m)(v + \delta v)$	M1 A1		
	$mv = mv + v\delta m + m\delta v - mg\delta t$ (to first order of δ terms) $\therefore 0 = m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t} - mg$	M1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = 3\lambda m$ $\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = g - 3\lambda v$	A1	4	
	Total		**8**	

MBM5 (cont)

Q	Solution	Marks	Total	Comments
8(a)	When displacement is x,			
	Tension = $\dfrac{\lambda x}{l} = \dfrac{4mn^2 a.x}{2a}$			
	$= 2mn^2x$	B1		
	Using $F = ma$,			
	$m\dfrac{d^2x}{dt^2} = mg - 2mn^2x - 2mn\dfrac{dx}{dt}$	M1 A1		
	$\dfrac{d^2x}{dt^2} + 2n\dfrac{dx}{dt} + 2n^2x = g$	A1	4	
(b)	CF $x = Ae^{pt}$			
	$p^2 + 2np + 2n^2 = 0$	M1		
	$p = \dfrac{-2n \pm \sqrt{4n^2 - 8n^2}}{2}$			
	$= -n \pm ni$	A1		
	$\therefore x = e^{-nt}(A\cos nt + B\sin nt)$	A1		
	PI $x = \dfrac{g}{2n^2}$	B1		
	$x = e^{-nt}(A\cos nt + B\sin nt) + \dfrac{g}{2n^2}$	M1		
	When $t = 0$, $x = 0$,			
	A $= -\dfrac{g}{2n^2}$	B1		
	$\dfrac{dx}{dt} = -n\,e^{-nt}(A\cos nt + B\sin nt) + e^{-nt}(-nA\sin nt + nB\cos nt)$	M1		
	When $t = 0$, $\dfrac{dx}{dt} = 0$			
	$\Rightarrow -nA + nB = 0$			
	$B = -\dfrac{g}{2n^2}$	A1		
	$x = \dfrac{g}{2n^2}\{1 - e^{-nt}(\cos nt + \sin nt)\}$	A1	9	
	Total		**13**	
	TOTAL		**60**	