General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 3

MBM3

Monday 20 June 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A cyclist travels along a straight horizontal road. She accelerates uniformly from a speed of $2\,\text{m s}^{-1}$ to $6\,\text{m s}^{-1}$ as she travels 10 metres.

(a) (i) Show that the acceleration of the cyclist is $1.6\,\text{m s}^{-2}$. *(2 marks)*

(ii) Find the time that it takes the cyclist to travel this distance. *(2 marks)*

(b) Model the cyclist as a particle of mass 65 kg. A constant resistance force of magnitude 35 N acts on the cyclist. A horizontal force of magnitude F newtons also acts on the cyclist in the direction of motion. Find F. *(3 marks)*

2 A car accelerates from rest along a straight road for 5 seconds. At time t seconds, its acceleration, $a\,\text{m s}^{-2}$, is given by

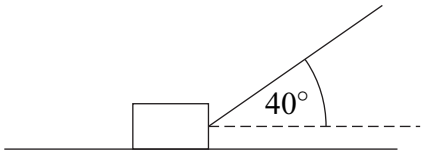
$$a = t - \frac{t^2}{5}$$

(a) By integrating, find an expression for the velocity of the car at time t. *(3 marks)*

(b) Find the velocity of the car at the end of the 5 second period. *(2 marks)*

(c) Find the distance travelled by the car during the 5 second period. *(4 marks)*

3 A box, of mass 50 kg, is dragged along rough horizontal ground by a rope attached to the box. The rope makes an angle of 40° with the ground and the tension in the rope is T newtons. The coefficient of friction between the box and the ground is 0.6. The diagram shows the box and the rope.



Model the box as a particle.

(a) Draw a diagram to show the forces acting on the box. *(1 mark)*

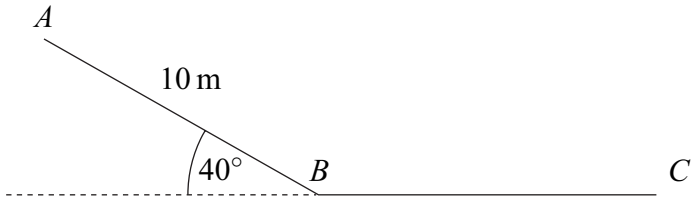
(b) Find, in terms of T, the magnitude of the normal reaction force acting on the box. *(3 marks)*

(c) Show that the magnitude of the friction force acting on the box is

$$294 - 0.6T\sin 40°$$ *(2 marks)*

(d) The box accelerates at $0.5\,\text{m s}^{-2}$. Find T. *(4 marks)*

4 The diagram shows a simple slide made of two sections AB and BC. Section AB has length 10 metres and is at an angle of 40° to the horizontal. Section BC is horizontal.



A child uses the slide, pushing himself off so that at A his speed is $2\,\text{m s}^{-1}$.

When he reaches B his speed is $6\,\text{m s}^{-1}$.

The mass of the child is 35 kg.

(a) (i) Calculate the kinetic energy of the child at A. *(2 marks)*

(ii) Calculate the gain in kinetic energy as the child moves from A to B. *(3 marks)*

(b) Show that the loss in potential energy as the child moves from A to B is 2200 joules, correct to three significant figures. *(2 marks)*

(c) As the child moves from A to B, a resistance force acts on him.

Assume that this force has a constant magnitude F newtons.

Find F. *(3 marks)*

(d) Assume that the same resistance force acts on the child as he moves on BC and that he stops at C. Find the length of BC. *(3 marks)*

5 An aeroplane is circling an airfield while waiting to land. At time t seconds, its position vector, $\mathbf{r}$ metres, with respect to an origin at the airfield is

$$\mathbf{r} = 500\sin(0.1t)\mathbf{i} + 500\cos(0.1t)\mathbf{j} + 1000\mathbf{k}$$

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively and the unit vector $\mathbf{k}$ is directed vertically upwards.

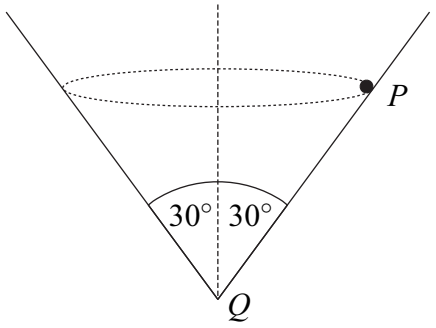
(a) Find the velocity of the aeroplane at time t. *(3 marks)*

(b) Hence show that the speed of the aeroplane is $50\,\text{m s}^{-1}$. *(4 marks)*

(c) Find the acceleration of the aeroplane. *(2 marks)*

(d) The mass of the aeroplane is 8000 kg. Find the magnitude of the resultant force acting on the aeroplane. *(3 marks)*

6 A particle P, of mass m, moves in a horizontal circular path on the inside of a smooth cone with vertex Q. The angle between PQ and the vertical is 30°. The diagram shows the cone and the path of the particle.



(a) Draw a diagram to show the forces acting on the particle. *(1 mark)*

(b) Show that the reaction force on the particle has magnitude $2mg$. *(2 marks)*

(c) The speed of the particle is v. Find the radius, r, of the circular path of P in terms of v and g. *(4 marks)*

(d) Explain what would happen to the radius if the speed were halved. *(2 marks)*

7 An elastic string has natural length 2 metres and modulus of elasticity 30 N. One end of the string is fixed at the point O. A particle of mass 0.15 kg is attached to the other end of the string. The particle is pulled down until it is 2.8 metres directly below O and released from rest.

(a) Calculate the elastic potential energy of the string just before the particle is released. *(2 marks)*

(b) After the particle has been released, it moves upwards.

(i) Show that the speed of the particle as it passes O is $3.02\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(4 marks)*

(ii) By considering the maximum height of the particle above O, show that the string does not become taut as the particle moves upwards. *(4 marks)*

8 At time $t = 0$, a boat has velocity $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ and is at the origin. After moving with a constant acceleration of $\mathbf{a}\,\mathrm{m\,s^{-2}}$ for 20 seconds, its position vector is $(60\mathbf{i} + 20\mathbf{j})$ metres. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find $\mathbf{a}$. *(3 marks)*

(b) Find the velocity of the boat at time t seconds. *(2 marks)*

(c) Find the speed of the boat when it is travelling south east. *(5 marks)*

END OF QUESTIONS