General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 2

MBM2

Monday 20 June 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A car accelerates from rest along a straight road for 5 seconds. At time t seconds, its acceleration, a m s^{-2}, is given by

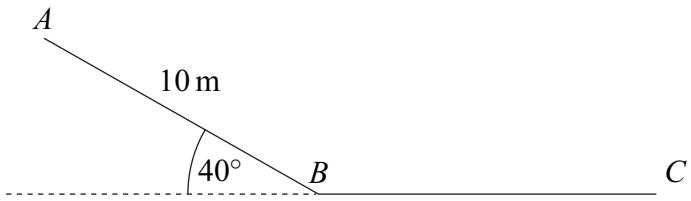
$$a = t - \frac{t^2}{5}$$

(a) By integrating, find an expression for the velocity of the car at time t. *(3 marks)*

(b) Find the velocity of the car at the end of the 5 second period. *(2 marks)*

(c) Find the distance travelled by the car during the 5 second period. *(4 marks)*

2 The diagram shows a simple slide made of two sections AB and BC. Section AB has length 10 metres and is at an angle of 40° to the horizontal. Section BC is horizontal.



A child uses the slide, pushing himself off so that at A his speed is 2 m s^{-1}.

When he reaches B his speed is 6 m s^{-1}.

The mass of the child is 35 kg.

(a) (i) Calculate the kinetic energy of the child at A. *(2 marks)*

(ii) Calculate the gain in kinetic energy as the child moves from A to B. *(3 marks)*

(b) Show that the loss in potential energy as the child moves from A to B is 2200 joules, correct to three significant figures. *(2 marks)*

(c) As the child moves from A to B, a resistance force acts on him.

Assume that this force has a constant magnitude F newtons.

Find F. *(3 marks)*

(d) Assume that the same resistance force acts on the child as he moves on BC and that he stops at C. Find the length of BC. *(3 marks)*

3 An aeroplane is circling an airfield while waiting to land. At time t seconds, its position vector, $\mathbf{r}$ metres, with respect to an origin at the airfield is

$$\mathbf{r} = 500\sin(0.1t)\mathbf{i} + 500\cos(0.1t)\mathbf{j} + 1000\mathbf{k}$$

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively and the unit vector $\mathbf{k}$ is directed vertically upwards.

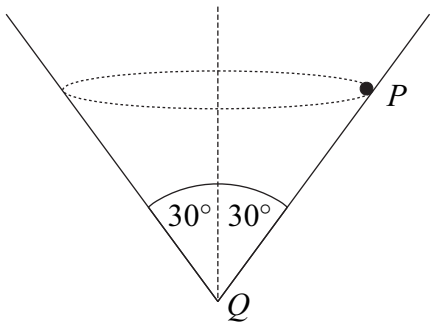
(a) Find the velocity of the aeroplane at time t. *(3 marks)*

(b) Hence show that the speed of the aeroplane is $50\,\mathrm{m\,s^{-1}}$. *(4 marks)*

(c) Find the acceleration of the aeroplane. *(2 marks)*

(d) The mass of the aeroplane is 8000 kg. Find the magnitude of the resultant force acting on the aeroplane. *(3 marks)*

4 A particle P, of mass m, moves in a horizontal circular path on the inside of a smooth cone with vertex Q. The angle between PQ and the vertical is 30°. The diagram shows the cone and the path of the particle.



(a) Draw a diagram to show the forces acting on the particle. *(1 mark)*

(b) Show that the reaction force on the particle has magnitude $2mg$. *(2 marks)*

(c) The speed of the particle is v. Find the radius, r, of the circular path of P in terms of v and g. *(4 marks)*

5 An elastic string has natural length 2 metres and modulus of elasticity 30 N. One end of the string is fixed at the point O. A particle of mass 0.15 kg is attached to the other end of the string. The particle is pulled down until it is 2.8 metres directly below O and released from rest.

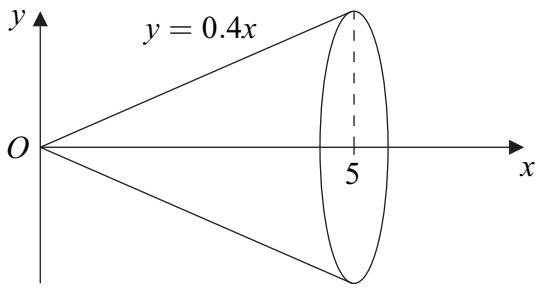
(a) Calculate the elastic potential energy of the string just before the particle is released. *(2 marks)*

(b) After the particle has been released, it moves upwards.

(i) Show that the speed of the particle as it passes O is $3.02\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(4 marks)*

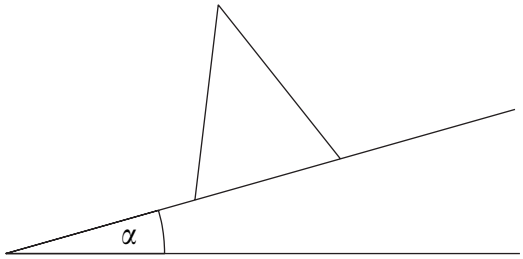
(ii) By considering the maximum height of the particle above O, show that the string does not become taut as the particle moves upwards. *(4 marks)*

6 A uniform **solid** cone is formed by rotating the finite region bounded by the lines with equations $y = 0.4x$, $y = 0$ and $x = 5$ through 360° about the x-axis. The cone is shown in the diagram below.



(a) Use integration to show that the distance of the centre of mass of the cone from the origin is $\frac{15}{4}$. *(5 marks)*

(b) The cone is placed on a rough slope, which is at an angle α to the horizontal, as shown in the diagram.



The angle α is gradually increased.

Assuming that the cone does not slide, find the value of α when the cone is on the point of toppling. *(5 marks)*

7 An elastic string has natural length l and modulus of elasticity λ. One end of the string is fixed and a particle of mass m is attached to the other end of the string.

(a) When the particle hangs vertically in equilibrium, the extension of the string is $\frac{l}{4}$. Show that $\lambda = 4mg$. *(2 marks)*

(b) The particle is then set into vertical motion and x is the displacement of the particle from the equilibrium position at time t. Assume that the string remains taut during the vertical motion.

(i) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{4gx}{l}$. *(5 marks)*

(ii) Hence find the period of the motion. *(2 marks)*

8 A stone, of mass m kg, is projected vertically upwards from a catapult. It leaves the catapult travelling at $20\,\mathrm{m\,s^{-1}}$. As it moves, it is subject to a resistance force of magnitude $0.7mv$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the stone at time t seconds.

At time t seconds, the height of the stone is x metres above the point of projection.

(a) Show that, as the stone is moving upwards,

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -0.7(v + 14)$$ *(2 marks)*

(b) Using the identity $\frac{v}{v+14} = 1 - \frac{14}{v+14}$, show that

$$x = \frac{10}{7}\left(20 - v + 14\ln\left(\frac{v+14}{34}\right)\right)$$ *(6 marks)*

(c) Hence find the maximum value of x. *(2 marks)*

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