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ASSESSMENT and QUALIFICATIONS ALLIANCE

# General Certificate of Education

# Mathematics and Statistics 6320
## *Specification B*

*MBD2 Discrete 2*

# Mark Scheme

## *2005 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key to Mark Scheme

| | | |
|---|---|---|
| **M** | mark is for | method |
| **m** | mark is dependent on one or more M marks and is for | method |
| **A** | mark is dependent on M or m marks and is for | accuracy |
| **B** | mark is independent of M or m marks and is for | accuracy |
| **E** | mark is for | explanation |
| **√or ft or F** | | follow through from previous incorrect result |
| **cao** | | correct answer only |
| **cso** | | correct solution only |
| **awfw** | | anything which falls within |
| **awrt** | | anything which rounds to |
| **acf** | | any correct form |
| **ag** | | answer given |
| **sc** | | special case |
| **oe** | | or equivalent |
| **sf** | | significant figure(s) |
| **dp** | | decimal place(s) |
| **A2,1** | | 2 or 1 (or 0) accuracy marks |
| **–*x* ee** | | deduct *x* marks for each error |
| **pi** | | possibly implied |
| **sca** | | substantially correct approach |

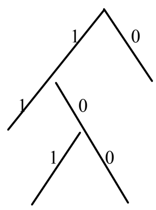
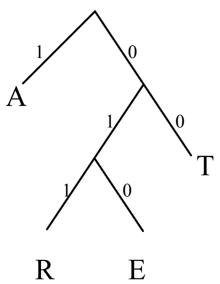
## Abbreviations used in Marking

| | |
|---|---|
| **MC – *x*** | deducted *x* marks for mis-copy |
| **MR – *x*** | deducted *x* marks for mis-read |
| **isw** | ignored subsequent working |
| **bod** | given benefit of doubt |
| **wr** | work replaced by candidate |
| **fb** | formulae book |

## Application of Mark Scheme

| | |
|---|---|
| **No method shown:** | |
| Correct answer without working | mark as in scheme |
| Incorrect answer without working | zero marks unless specified otherwise |
| | |
| **More than one method / choice of solution:** | |
| 2 or more complete attempts, neither/none crossed out | mark both/all fully and award the mean mark rounded down |
| 1 complete and 1 partial attempt, neither crossed out | award credit for the complete solution only |
| | |
| **Crossed out work** | do not mark unless it has not been replaced |
| | |
| **Alternative solution** using a correct or partially correct method | award method and accuracy marks as appropriate |

## Mathematics and Statistics B Discrete 2 MBD2 June 2005

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)(i)** | There are vertices of odd degree, so the network is not Eulerian. | B1 | 1 | |
| **(ii)** | Odd vertices: *A B F H* | M1 | | |
| | Pairings *AB FH*: adds 80+60 | A1 | | |
| | *AF BH*: adds >>140 | | | |
| | *AH BF*: adds >>140 | A1 | | |
| | So shortest route has length 1100 + 140 = 1240 metres. | B1 | 4 | |
| **(iii)** | e.g.*AHFDBACGHFBCEGFEBA* | M1A1 | | |
| | | A1 | 3 | |
| **(b)(i)** | *ACEGHFDBA* | M1A1 | | |
| | 50+50+50+50+60+70+60+80=470 | A1 | 3 | |
| **(ii)** | *AH* + seven other arcs each ≥50 | B1 | 1 | |
| **(iii)** | Only two footpaths out of *A* are *AB, AC.* Similarly for *D* and *H.* | B1 | | |
| | Then the only way to complete the route via *E* is with *CE, EG.* | B1 | 2 | |
| **(iv)** | (ii) the route of 470 beats any involving *AH.* | M1 | | |
| | By (iii) the route of 470 is the *only* one not using *AH* and hence is the shortest. | A1 | 2 | |
| | **Total** | | **16** | |
| **2(a)** | 111101100 | B1 | 1 | |
| **(b)** | REAR | B1 | 1 | |
| **(c)** |  | M1<br>A1 | 2 | |
| **(d)(i)** | T=00 | B1 | | |
| | T is at start and finish and T=0 ⇒ word starts TT | B1 | 2 | |
| **(ii)** |  | M1<br>A1 | 2 | |
| | **Total** | | **8** | |

**MBD2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | Aux: $M^2 - M - 2 = 0$, roots 2, –1<br>So comp function is $A.2^n + B.(-1)^n$<br>Particular solution $u_n = -½$ gives<br>$(-½) - (-½) - 2.(-½) - 1$<br>So general solution is these two added. | M1A1<br>A1<br>M1<br>A1 | 5 | (or substitute given answer into the relation) |
| **(b)(i)** | $u_0=0 \Rightarrow A + B - ½ = 0$<br>$u_1=1 \Rightarrow 2A - B - ½ = 1$<br>Solving gives A = 2/3, B= –1/6<br>and $u_n = (2/3)2^n - (-1)^n/6 - ½$ | <br>M1<br>A1<br>A1 | 3 | |
| **(ii)** | $u_n = 2^{n+1}/3 - 1/6 - 1/2$ ($n$ even)<br>$u_n = 2^{n+1}/3 + 1/6 - 1/2$ ($n$ odd)<br>So the integer $u_n$ is $2^{n+1}/3$ with its fractional bit removed. | B1<br>B1<br>B1 | 3 | |
| | **Total** | | **11** | |
| **4(a)** | 000000<br>e.g. 001110 + 110011 = 111101<br>and 110011 + 100010 = 010001 | B1<br>M1<br>A1 | 3 | |
| **(b)** | $3^{rd} = 4^{th}$<br>$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$ | B1<br>M1<br>A1√ | 3 | ft |
| **(c)** | matrix × $(0\ 1\ 0\ 0\ 0\ 1)^T = (0\ 0\ 0)^T$<br>matrix × $(1\ 0\ 1\ 0\ 1\ 0)^T = (1\ 0\ 1)^T$ ← 3rd column<br>so first half correct and second half should be 100010. | M1<br>A1<br>B1 | 3 | |
| | **Total** | | **9** | |

**MBD2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | e.g. *S A B T* 4<br>*S A C B T* 2<br>*S C D T* 2<br>*S E D F T* 3<br>*S E F T* 3 (total 14) | M1<br>A1<br>A1<br>A1<br>A1 | <br><br><br><br>5 | <br>Any two paths<br>} Remaining paths to make total 14<br>(or 2 for cao) |
| **(b)** | *EF, ED, CD, BT*<br>(3 + 3 + 2 + 6 = 14) | M1 A1 | 2 | |
| **(c)(i)** | Flow in *SE* ≤ flow out at *E* ≤ 3+3 | B1 | 1 | |
| **(ii)** | Flow into *C* = flow out of *C* ≤ 3+2 = 5<br>So if *AC* has a flow of 4 then *SC* has a flow of at most 1. | M1<br>A1 | <br>2 | |
| **(iii)** | If *AC* is saturated the maximum flows in *SA, SC* and *SE* are 6 (its capacity), 1 (by part (ii)) and 6 (by part (i)). So if *AC* is saturated the maximum flow out of *S* is 6+6+1=13 which is less than the unrestricted maximum flow. | M1<br><br>A1 | <br><br>2 | |
| | **Total** | | **12** | |

**MBD2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $p_2 = 1,\ p_3 = 3,\ p_4 = 6$ | B1 B1 | 2 | |
| **(b)** | 4 new points created, hence | | | |
| | $p_5$ = existing points + 4 | B1 | | |
| | Similarly when the $n$th line is added it | | | |
| | adds $n$-1 new points. | B1 | 2 | |
| **(c)** | $p_n = p_{n-1} + (n - 1)$ | M1 | | |
| | $= p_{n-2} + (n-2) + (n-1)$ | | | (or direct verification by substitution) |
| | … | | | |
| | $= p_1 + 1 + 2 + \ldots + (n-1)$ | A1 | | |
| | $= 0 + 1 + 2 + \ldots + (n-1)$ | | | |
| | $= \frac{1}{2} n(n - 1)$ | A1 | 3 | |
| | **Total** | | **7** | |
| **7(a)** | Maximise $P = 15x + 10y + 10z$ | B1 | | |
| | Subject to | | | |
| | $(x \geq 0 \quad y \geq 0 \quad z \geq 0)$ | | | |
| | $x + 2y + z \leq 60$ | B1 | | one inequality |
| | $x + y + z \leq 55$ | | | |
| | $3x + 6y + 2z \leq 140$ | B1 | 3 | the two others |
| **(b)** | 1 -15 -10 -10 0 0 0 0 | | | |
| | 0 1 2 1 1 0 0 60 | B1 | | coefficients of $x,y,z$ |
| | 0 1 1 1 0 1 0 55 | | | |
| | 0 3 6 2 0 0 1 140 | B1 | 2 | appropriate slacks |
| **(c)** | 1 -5 0 0 0 10 0 550 | M1A1 | | pivot |
| | 0 0 1 0 1 -1 0 5 | | | |
| | 0 1 1 **1** 0 1 0 55 | M1A1 | | row operations |
| | 0 1 4 0 0 -2 1 30 | A1 | 5 | |
| **(d)** | 1 0 20 0 0 0 5 700 | | | |
| | 0 0 1 0 1 -1 0 5 | M1A1 | | |
| | 0 0 -3 1 0 3 -1 25 | M1A1 | 4 | |
| | 0 **1** 4 0 0 -2 1 30 | | | |
| **(e)** | Maximum profit £700 | B1√ | | ft |
| | Make 30 Xtrafast, 0 Yourelax, 25 Zizz | B1√ | | ft |
| | e.g. Not helpful for people who want Yourelax. | B1 | 3 | |
| | **Total** | | **17** | |
| | **TOTAL** | | **80** | |