GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBS7)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

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Question Number and Part	Solution	Marks	Total	Comments
1	H_0: $\mu_A - \mu_B = 0.5$	B1		allow 0, rather than 0.5, in H_0 must be population means
	H_1: $\mu_A - \mu_B \neq 0.5$	B1		must include 0.5 in H_0 & H_1
	SL $\alpha = 0.05$ (5%) CV $z = 1.96$	B1		cao: (allow 1.64 to 1.65 awfw for '>' in H_0)
	$\bar{x}_A = 3.44$ $\bar{x}_B = 2.76$ $\sigma = 0.4$			
	$z = \dfrac{(\bar{x}_A - \bar{x}_B) - \mu_0}{\sqrt{\dfrac{\sigma^2}{n_A} + \dfrac{\sigma^2}{n_B}}}$	M1		use of; allow no μ_0
	$= \dfrac{(3.44 - 2.76) - 0.5}{\sqrt{\dfrac{0.4^2}{20} + \dfrac{0.4^2}{25}}}$	A1		allow $\mu_0 = 0$
		A1		cao
	= 1.49 to 1.51	A1		awfw (ca = 1.5) (a = 5.67 with $\mu_0 = 0$)
	Thus, no evidence, at 5% level, to reject claim (that $\mu_A - \mu_B = 0.5$)	A1√	8	or equivalent ft on z and CV
	Total		**8**	

MBS7 (cont)

Question Number and Part	Solution	Marks	Total	Comments
2(a)	H_0: $\lambda = 8$ (or $p = 0.008$) H_1: $\lambda < 8$ (or $p < 0.008$)	B1		both; no mixtures of λ & p
	$P(X \leq 3 \mid \text{Po}(8))$	M1		use of Po(8)
	= 0.042 to 0.043 (< 5%)	A1		awfw; (ca = 0.0424)
	Thus evidence, at 5% level, that average number (of faulty bottles per batch) has decreased	A1√	4	or equivalent ft on probability versus 5%
(b)(i)	$\bar{x} = \frac{\sum fx}{250} = \frac{500}{250}$	B1	1	cao ratio; (**ag** of 2)
(ii)	H_0: $X \sim$ Poisson H_1: not H_0	B1		at least H_0
	x O p E 0 41 0.1353 33.825 1 57 0.2707 67.675 2 74 0.2707 67.675 3 35 0.1804 45.100 4 28 0.0902 22.550 5 12 0.0361 9.025) 6 3 0.0121 3.025) ≥7 0 0.0045 1.125) T 200 1.0000 250.000	M1 M1 M1		attempted Poisson probabilities with $\lambda = 2$ attempt at $E = 250 \times p$ attempt at ≥7 (may be implied) attempt at combining (13.175)
	$\chi^2 = \Sigma \frac{(O-E)^2}{E}$	M1		use of
	= 7.50 to 7.75	A1		awfw
	SL α = 0.01 (1%) DF ν = 4	B1		cao
	CV χ^2 = 13.277 or CV χ^2 = 15.086 (ν = 5)			awfw 13.2 to 13.3 awfw 15.0 to 15.1
	Thus no evidence, at 1% level, to reject hypothesis that distribution is Poisson	B1 A1√	10	or equivalent ft on χ^2 and CV
	Total		**15**	

MBS7 (cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	$\sum x = 140 \quad \sum x^2 = 3500 \quad \sum xy = 1587$ $\sum y = 63 \quad \sum y^2 = 722.9 \quad \bar{x} = 20 \quad \bar{y} = 9$ $S_{xx} = 700 \quad S_{yy} = 155.9 \quad S_{xy} = 327$			
	$\hat{\beta} = 0.467 \qquad \hat{\alpha} = -0.343$	B1 B1	2	awrt
(b)(i)	$RSS = 155.9 - \dfrac{327^2}{700}$	M1		use of; even if called s^2
		M1		use of $RSS \div 5$
	$s^2 = \dfrac{RSS}{5} =$ 0.628 to 0.630	A1	3	awfw
(ii)	H_0: $\beta = 0.5$			
	H_1: $\beta \neq 0.5$	B1		both
	SL $\alpha = 0.05(5\%)$			
	DF $\nu = 7 - 2 = 5$	B1		cao
	CV $\lvert t \rvert = 2.571$	B1		awrt 2.57; ignore sign
	$t = \dfrac{\hat{\beta} - \beta_0}{\sqrt{\dfrac{s^2}{S_{xx}}}} = \dfrac{0.467 - 0.5}{\sqrt{\dfrac{0.629}{700}}} =$ –1.11 to –1.09	M1 A1		use of awfw; ignore sign
	Thus no evidence, at 5% level, that value of β is not 0.5	A1√	6	or equivalent ft on t and CV – consistent signs
(c)(i)	$y = -0.343 + 0.467 \times 45 =$ 20.5 to 20.9	B1	1	awfw; (allow 22.1 to 22.3 awfw for use with $\beta = 0.5$)
(ii)	$x = 45 \Rightarrow$ half-way across/middle	E1	1	or equivalent (eg 90/2)
(iii)	*Statistical*: 45 is outside observed range	B1		or equivalent
	Practical: Maximum depth unlikely to bein middle of river **or** Riverbed is unlikely to be V-shaped	E1	2	or equivalent or sensible alternative
	Total		**15**	
4	$T \sim E(2)$			
(a)	1	B1	1	cao; accept 'unity'
(b)	$P(S > 5) = P(T > 4)$	B1		4 cao
	$= 1 - \left(1 - e^{-\frac{4}{2}}\right) = e^{-2}$	M1		use of exponential cdf or pdf with $\lambda = 0.5$ or 2
	$= 0.135$	A1	3	awrt
(c)	$P(S < 5 \mid S > 3) = P(T < 4 \mid T > 2)$	M1		use of conditional probability
	Exponential has 'no memory' so	M1		use of; may be implied
	$= P(T < 2)$	A1		2 cao; (even from 5 – 3)
	$= 1 - e^{-1} = 0.632$	A1	4	awrt
(d)	Probability = $(b)^5 = 0.000044$ to 0.000046	B1√		awfw; ft on (b)
	Implies an extremely rare event so casts doubt on model	E1	2	rare event, or equivalent **ag**
	Total		**10**	

MBS7 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	Mean = 20 + 10 + 75 + 10 = 115	B1		cao
	Variance = $3^2 + 3^2 + 10^2 + 2^2$	M1		adding variances
	= 122	A1		cao; (σ = 11.0 to 11.1 awfw) (M0 A0 for σ = 18)
	$P(J < 120) = P\left(Z < \frac{120-115}{\sqrt{122}}\right) =$	M1		standardising 120 using ft (μ & σ)
	$P(Z < 0.453)$ = 0.673 to 0.677	A1	5	awrt
(b)(i)	$P\left(R > \frac{2}{3}J\right) = P(3R > 2J)$	M1		use of $\frac{2}{3}$ or 3 & 2 and sum of 4 parts
	$= P(3R > 2(C + T + R + W))$			
	$= P(R > 2(C + T + W))$	m1	2	cancelling of $2R$
	$\Rightarrow$ answer			**ag**
(ii)	Mean = 75 – 2(20 + 10 + 10) = **–**5	B1		cao; ignore sign
	Variance = $10^2 + 2^2(3^2 + 3^2 + 2^2)$	M1		using variances and $(-2)^2$ or +4
	= 188	A1		cao; (σ = 13.7 awrt)
	$P(X > 0) = P\left(Z > \frac{0-(-5)}{\sqrt{188}}\right) =$	M1		standardising 0 using ft (μ & σ)
	$P(Z > 0.36466) = 1 - \Phi(0.36466)$			
	= 0.355 to 0.360	A1	5	awfw
	Total		**12**	
	TOTAL		**60**	