GCE 2005
January Series

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP6)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
-*x* EE		deduct *x* marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
SF		significant figure(s)
DP		decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working .... mark as in scheme

Incorrect answer without working .... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

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Question Number and Part	Solution	Marks	Total	Comments
1	Aux. eqn. is $4m^2 - 8m + 5 = 0$	B1		
	Solving: $m = 1 \pm \frac{1}{2}\mathrm{i}$	M1 A1		
	G.S. is $y = \mathrm{e}^x (A \cos\frac{1}{2}x + B \sin\frac{1}{2}x)$	B1√		
		B1√	5	Give one B1 only for real roots followed through correctly
	Total		**5**	
2(a)	$\int (\cosh x + \operatorname{sech}^2 x)\,\mathrm{d}x$	M1		Ignore limits until final answer
	$= \sinh x + \tanh x$	A1 A1		
	$= 1.35$	A1	4	
	Total		**4**	
3(a)(i)	$\lvert \pm (u - v) \rvert$	B1		
(a)(ii)	$\arg u - \arg v$	B1	2	
(b)	Clearly indicated parallelogram with W at end of main diagonal **or** Vector triangle with sides **u** , **v** , **w**	B1	1	
	Total		**3**	
4(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - \frac{1}{t^2}$	B1 B1		
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \frac{4}{t^2} + 1 - \frac{2}{t^2} + \frac{1}{t^4}$	M1		
	$= \left(\frac{t^2+1}{t^2}\right)^2$	A1	4	Legitimately shown
(b)	$S = 2\pi \int \left(\frac{t^2+1}{t}\right)\left(\frac{t^2+1}{t^2}\right)\mathrm{d}t$	B1		
	$= 2\pi \int \left(\frac{t^4+2t^2+1}{t^3}\right)\mathrm{d}t$	M1		Helpful simplification
	$= 2\pi \int \left(t + \frac{2}{t} + \frac{1}{t^3}\right)\mathrm{d}t$	A1		Suitable form for integrating
	$= 2\pi \left[\frac{1}{2}t^2 + 2\ln t - \frac{1}{2t^2}\right]$	A1√ A1√		for the log term for the other (two) terms
	$= \pi \left[\frac{15}{4} + 4\ln 2\right]$	A1	6	cao (any correct exact form)
	Total		**10**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	$LHS \equiv 2\left(\frac{1}{2}\left[e^{x} - e^{-x}\right]\right)\left(\frac{1}{2}\left[e^{x} + e^{-x}\right]\right)$	M1		
	$\equiv \frac{1}{2}\left[e^{2x} - e^{-2x}\right] \equiv \sinh 2x \equiv RHS$	A1	2	
(b)	I.F. is $\exp\left\{\int \tanh x \ \mathrm{d}x\right\}$	B1		
	$= \exp\left\{\ln(\cosh x)\right\} = \cosh x$	M1 A1		
	Then d.e. becomes			
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y \cosh x\right) = \frac{1}{2}\sinh 2x$	B1		RHS in integrable form
	$\int RHS = \frac{1}{4}\cosh 2x$ or $\frac{1}{2}\cosh^2 x$ etc.	A1√		
	Use of $x = 0$, $y = 1$ to find const. of $\int$	M1		
	$y \cosh x = \frac{3}{4} + \frac{1}{4}\cosh 2x$	A1	7	Including fully correct solution A0 for correct C found from incorrect division by cosh x.
	Total		**9**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$\frac{2t}{1+t^2}.t+\frac{2(1+t^2)}{1-t^2}$	M1		Use of correct half-angle forms for sin x and cos x
	$=\frac{2t^2(1-t^2)+2(1+t^2)^2}{(1+t^2)(1-t^2)}$			
	$=\frac{2+6t^2}{(1+t^2)(1-t^2)}$	A1	2	Answer given
(b)	$4+12t^2+5-5t^4=0$	M1		Polynomial attempt
	$5t^4-12t^2-9=0$	A1		
	(Since $t^2>0$) $t^2=3$	B1√		ft (the) positive root for t^2
	$\tan\frac{1}{2}x=\pm\sqrt{3}$	M1		Including attempt to solve for x
	$x=\frac{2\pi}{3},\ \frac{4\pi}{3}$ (decimals, in radians, OK)	A1√ A1	6	ft first answer For both A's, two correct answers + no extras
(c)	**(i)** $\int\frac{3(1-t^2)(1+t^2)}{2+6t^2}.\frac{2\,\mathrm{d}t}{1+t^2}$	M1 B1		Complete substn. method dx in terms of t's
	$=\int\frac{3-3t^2}{1+3t^2}\,\mathrm{d}t$	A1		
	Upper limit = $\frac{1}{\sqrt{3}}$	B1	4	
	(ii) $=\int\left(\frac{4}{1+3t^2}-1\right)\mathrm{d}t$	B1		Separated into integrable bits
	$=-t+\frac{4}{\sqrt{3}}\tan^{-1}(t\sqrt{3})$	M1 A1		Must be arctan for the M
	$=\frac{\pi-1}{\sqrt{3}}$	A1	4	cao
	Total		**16**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	$(1 + \mathrm{i}\tan\theta)^3$ expanded	M1		Multn. or binomial theorem
	Re. part $= 1 - 3\tan^2\theta$	A1	2	Ignore Im. parts
(b)	$(1 + \mathrm{i}\tan\theta)^3 = \left(\dfrac{\cos\theta + \mathrm{i}\sin\theta}{\cos\theta}\right)^3$	B1		
	$= \left(\dfrac{\cos 3\theta + \mathrm{i}\sin 3\theta}{\cos^3\theta}\right)$	M1		Use of de Moivre's theorem
	Equating Re. parts $\Rightarrow$ $1 - 3\tan^2\theta = \dfrac{\cos 3\theta}{\cos^3\theta}$	A1	3	
	Total		**5**	
8(a)	Char. Eqn. is $\lambda^2 - 9\lambda + 8 = 0$	M1 A1		
	$\lambda = 1$ or 8	A1		
	$\lambda = 1 \Rightarrow 2x + 5y = 0$	M1		Either eval. substd. back
	gives eigenvectors $p\begin{pmatrix}5\\-2\end{pmatrix}$	A1		
	$\lambda = 8 \Rightarrow -5x + 5y = 0$			
	gives eigenvectors $q\begin{pmatrix}1\\1\end{pmatrix}$	A1	6	Any non-zero p, q will serve
(b)(i)	$x = 2,\ y = -1$ substd. in x' & y'	M1		Both x' *and* y' eqns.
	to get $x' = 2,\ y' = -1$	A1	2	
(ii)	$\begin{pmatrix}x'-2\\y'+1\end{pmatrix} = \begin{pmatrix}3 & 5\\2 & 6\end{pmatrix}\begin{pmatrix}x-2\\y+1\end{pmatrix}$	B1	1	i.e. $\alpha = 2, \beta = -1$
(iii)	$2(x-2) + 5(y+1) = 0$ **Or** $2x + 5y + 1 = 0$ or equivalent	B2,1√	2	Give B1 for $2x + 5y = 0$ or ft from their eval. of 1
(c)	E.g. $x' = 3x + 5(x + c) + 1 = 8x + 5c + 1$	B1		Use of $y = x + c$ at any stage
	$y' = 2x + 6(x + c) + 1 = 8x + 6c + 1$	M1		
	$= x' + c$	A1	3	
	Total		**14**	

MBP6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
9(a)	$I_n = \left[\frac{1}{3}e^{3x}\tan^n x\right] -$	M1 A1		
	$\int \frac{1}{3}e^{3x} n.\tan^{n-1} x.\sec^2 x\,dx$	A1		
	$\Rightarrow 3I_n =$ $\left(\sqrt{3}\right)^n.e^{\pi} - n\int e^{3x}\tan^{n-1}x(1+\tan^2 x)\,dx$	M1		Use of $\sec^2 = 1 + \tan^2$ to get I_{n-1} and I_{n+1} involved
	$= \left(\sqrt{3}\right)^n.e^{\pi} - n\left\{I_{n-1} + I_{n+1}\right\}$			
	$\Rightarrow nI_{n+1} + 3I_n + nI_{n-1} = \left(\sqrt{3}\right)^n.e^{\pi}$	A1	5	Answer given
	<u>ALTERNATIVE</u>:			
	$I_n = \int e^{3x}\tan^{n-2}x(\sec^2 x - 1)\,dx$	(M1)		
	$= \left[e^{3x}\frac{\tan^{n-1}x}{n-1}\right] - \int\frac{\tan^{n-1}x}{n-1}.3e^{3x}\,dx - I_{n-2}$	(A1) (A1)		
	$\Rightarrow (n-1)(I_n + I_{n-2}) = \left(\sqrt{3}\right)^{n-1}.e^{\pi} - 3I_{n-1}$	(M1)		
	$\Rightarrow$ result (one step down)	(A1)		
(b)(i)	$n = 1 \Rightarrow I_2 + 3I_1 + I_0 = \left(\sqrt{3}\right)e^{\pi}$	B1		
	$n = 3 \Rightarrow 3I_4 + 3I_3 + 3I_2 = \left(\sqrt{3}\right)^3 e^{\pi}$	B1		
	Thus $I_4 + I_3 + I_2 = \sqrt{3}\,e^{\pi} = I_2 + 3I_1 + I_0$	M1 A1		
	$\Rightarrow I_4 + I_3 - 3I_1 = I_0$	A1	5	Answer given
(b)(ii)	Use of $\sec^2 x = 1 + \tan^2 x$ to get	M1		
	$I = I_4 + I_3 - 3I_1$	A1		
	$I_0 = \int e^{3x}dx$	M1		Attempt to integrate this
	$= \frac{1}{3}\left(e^{\pi} - 1\right)$	A1	4	
	Total		**14**	
	TOTAL		**80**	