General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP5**
Unit Pure 5

Thursday 27 January 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve is given by the equation

$$y = x^2 - e^x$$

(a) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

(b) Show that $\dfrac{d^4y}{dx^4} = \dfrac{d^3y}{dx^3}$ for all values of x. *(1 mark)*

(c) Show that the curve has only one point of inflection and find its coordinates. *(4 marks)*

2 Use Simpson's rule with five ordinates (four equal strips) to find an approximation to the integral

$$\int_0^2 \sqrt{1 + x^2}\, dx$$

giving your answer to three decimal places. *(4 marks)*

3 Find the general solution, in radians, of the equation

$$\sin 2x + \cos x = 0$$ *(5 marks)*

4 (a) (i) Show that $(2 - x)^{-2}$ can be written as $\dfrac{1}{4}\left(1 - \dfrac{x}{2}\right)^{-2}$. *(1 mark)*

(ii) Obtain the first three terms of the binomial expansion of $(2 - x)^{-2}$ in ascending powers of x. *(3 marks)*

(iii) State the range of values of x for which the full expansion is valid. *(2 marks)*

(b) Using the substitution $u = 2 - x$, or otherwise, find $\displaystyle\int_0^{\frac{1}{2}} \frac{x}{(2 - x)^2}\, dx$ in the form $k - \ln\dfrac{4}{3}$, where k is a constant to be found. *(6 marks)*

5 A curve has equation $y = \dfrac{x^2 - 6}{2x - 5}$.

(a) Prove that there are no real values of x for which $2 < y < 3$. *(6 marks)*

(b) Hence find the coordinates of the two turning points on the curve. *(3 marks)*

(c) (i) State the equation of the vertical asymptote to the curve. *(1 mark)*

(ii) Given that $x^2 - 6 \equiv (2x - 5)\left(\frac{1}{2}x + \frac{5}{4}\right) + \frac{1}{4}$, find the equation of the oblique asymptote to the curve. *(2 marks)*

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

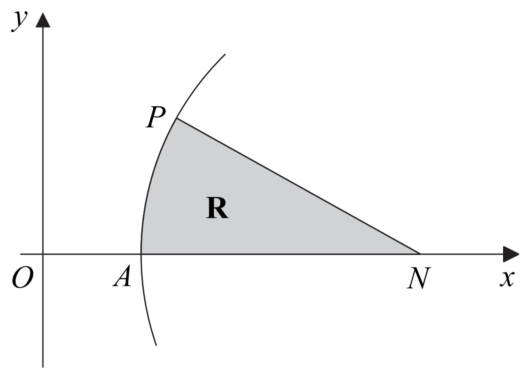
The line l_2 has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$.

(a) Show that the lines l_1 and l_2 intersect and find the position vector of their point of intersection. *(4 marks)*

(b) The lines l_1 and l_2 lie in the plane Π. Write down a vector equation of Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

7 The diagram shows part of a curve.



This curve is defined parametrically by

$$x = 4t + \frac{1}{t}, \qquad y = 4t - \frac{1}{t}, \qquad t > 0$$

The point P on the curve is where $t = 1$.
The normal to the curve at P intersects the x-axis at N.
The curve cuts the positive x-axis at the point A.

(a) Show that $t = \frac{1}{2}$ at the point A. *(2 marks)*

(b) Show that $\frac{dy}{dx} = \frac{4t^2 + 1}{4t^2 - 1}$. *(2 marks)*

(c) (i) Find an equation of the normal PN. *(3 marks)*

(ii) Hence show that the x-coordinate of N is 10. *(1 mark)*

(d) (i) Express $x + y$ and $x - y$ in terms of t. *(2 marks)*

(ii) Hence find a cartesian equation for the curve. *(1 mark)*

(e) The region **R**, bounded by the curve, the normal PN and the x-axis, is shown shaded in the diagram. Using your answer to part (d)(ii) and given that the area of **R** is $15 - 8\ln 2$, find the exact value of $\int_4^5 \sqrt{x^2 - 16}\, dx$. *(3 marks)*

END OF QUESTIONS