MATHEMATICS AND STATISTICS   MBP4
(SPECIFICATION B)
Unit Pure 4

Tuesday 25 January 2005   Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MBP4

Answer **all** questions.

1 A curve has equation

$$y = \frac{4x}{\sin x}, \quad 0 < x < \pi$$

(a) Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \dfrac{\pi}{2}$. *(4 marks)*

(b) Use the result from part (a) to find the approximate change in y when x increases from $\dfrac{\pi}{2}$ to $\dfrac{\pi}{2} + 0.01$. *(2 marks)*

2 The polynomial p(x) is given by p(x) $= x^3 - 3x + 2$.

(a) Find the remainder when p(x) is divided by $x + 1$. *(2 marks)*

(b) Given that $x + 2$ is a factor of p(x), express p(x) as a product of linear factors. *(3 marks)*

(c) Simplify the following algebraic fraction as far as possible:

$$\frac{x^3 - 3x + 2}{x^2 - 1}$$ *(2 marks)*

3 A circle has equation $x^2 + y^2 - 6x + 10y + 18 = 0$.

(a) (i) Find the coordinates of the centre, C. *(2 marks)*

(ii) Find the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x-axis. *(2 marks)*

(c) The point P has coordinates $(11, 6)$. Find the length of CP and hence find the length of a tangent from P to the circle. *(3 marks)*

4 (a) (i) Express $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.   *(1 mark)*

(ii) Express $\cos 2\theta$ in terms of $\cos \theta$.   *(1 mark)*

(iii) Prove the identity

$$\tan \theta (1 + \cos 2\theta) \equiv \sin 2\theta$$ *(2 marks)*

(b) Solve the equation

$$\tan \theta (1 + \cos 2\theta) = 2 \sin^2 2\theta$$

for $0 < \theta < \pi$, leaving your answers in terms of π.   *(6 marks)*

5 (a) Solve the equation $3^x = 7$, giving your answer to three significant figures.   *(2 marks)*

(b) (i) Sketch the graphs of $y = 3^x$ and $y = 7 - x^2$ on the same axes.   *(2 marks)*

(ii) Hence state the number of roots of the equation $3^x = 7 - x^2$.   *(1 mark)*

(c) (i) Show that the equation $3^x = 7 - x^2$ can be written in the form

$$x = \frac{\ln(7 - x^2)}{\ln 3}$$ *(1 mark)*

(ii) Use the iterative formula

$$x_{n+1} = \frac{\ln(7 - x_n^2)}{\ln 3}, \quad x_1 = 1.5$$

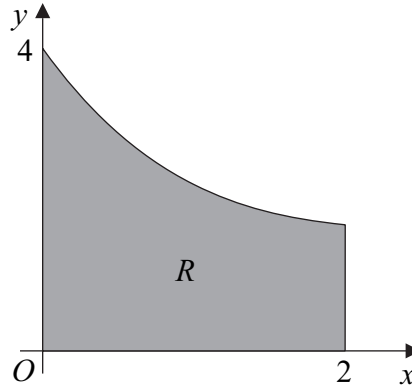to find the values of x_2 and x_3, giving your answers to three decimal places.   *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

6 A curve has equation $y = 1 + \dfrac{6}{3x + 2}$ and is sketched below for $0 \leqslant x \leqslant 2$.

The shaded region R is bounded by the curve, the coordinate axes and the line $x = 2$.

(a) Express y^2 in the form $1 + \dfrac{A}{3x + 2} + \dfrac{B}{(3x + 2)^2}$. *(2 marks)*

(b) Find:

    (i) $\displaystyle\int \dfrac{1}{3x + 2}\,dx$; *(2 marks)*

    (ii) $\displaystyle\int \dfrac{1}{(3x + 2)^2}\,dx$. *(2 marks)*

(c) Find the volume of the solid formed when R is rotated completely about the x-axis, giving your answer to three significant figures. *(5 marks)*

7 (a) The function f is defined for $-\dfrac{\pi}{6} < x < \dfrac{\pi}{6}$ by $f(x) = \tan 3x$.

The graph of $y = f(x)$ is sketched below.
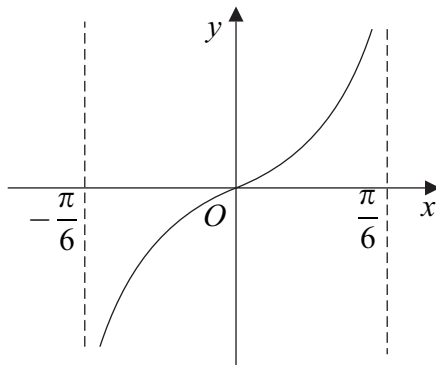
The inverse of f is f^{-1}.

   (i) Sketch the graph of $y = f^{-1}(x)$.   *(2 marks)*

   (ii) Find an expression for $f^{-1}(x)$.   *(2 marks)*

(b) A curve has equation $x = \tan 3y$.

   (i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.   *(2 marks)*

   (ii) Hence find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $y = \dfrac{\pi}{9}$.   *(3 marks)*

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE