GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP4)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

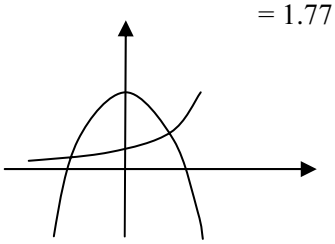
Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

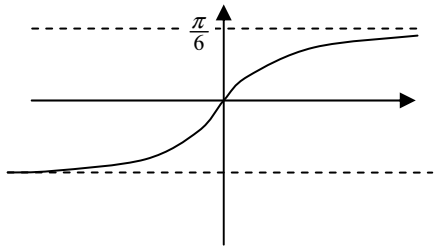
Mathematics and Statistics B Pure 4 MBP4 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$\frac{\mathrm{d}(\sin x)}{\mathrm{d}x} = \cos x$	B1		or $\frac{\mathrm{d}(\mathrm{cosec}\, x)}{\mathrm{d}x} = -\mathrm{cosec}\, x \cot x$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\sin x - 4x\cos x}{(\sin x)^2}$	M1		quotient rule, or product rule, attempt (condone sign errors)
	(oe $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\,\mathrm{cosec}\, x - 4x\,\mathrm{cosec}\, x \cot x$)	A1		correct
	$x = \frac{\pi}{2} \quad \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	A1√	4	
(b)	$\delta y \approx \frac{\mathrm{d}y}{\mathrm{d}x} \times \delta x$	M1		Stated or used – but NOT $\frac{\mathrm{d}y}{\mathrm{d}t} = \ldots$ etc
	$= 0.04$	A1√	2	ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $\delta x = 0.01$ Correct answer may score M0
	Total		**6**	
2(a)	p(–1) = –1+3+2	M1		p(–1) = ... or long division to remainder
	(remainder) = 4	A1	2	
(b)	Attempt at quadratic or long division	M1		or another linear factor
	$(x+2)(x^2-2x+1)$	A1		Or $(x + 2)(x – 1)$
	$= (x+2)(x-1)^2$	A1	3	
(c)	$\frac{(x+2)(x-1)^2}{(x+1)(x-1)}$	M1		Their (b) and denominator factorised
	$= \frac{(x+2)(x-1)}{(x+1)}$	A1	2	Withhold if further incorrect cancelling
	Total		**7**	
3(a)(i)	$(x-3)^2 + (y+5)^2$	M1		Attempt at completing square (generous) or one coordinate correct
	Centre (3, –5)	A1	2	
(ii)	$r^2 = 9 + 25 - 18 = 16$	M1		3 numbers - condone sign error
	$r = 4$	A1	2	
(b)	$\lvert y_C \rvert > 4$	E1		Or no real roots when $y = 0$
	Centre below x-axis	E1	2	
(c)	$CP^2 = 8^2 + 11^2 \quad \Rightarrow CP = \sqrt{185}$	B1√		ft their C
	$PT^2 = CP^2 - r^2$	M1		
	$PT^2 = 185 - 16 = 169 \quad \Rightarrow PT = 13$	A1√	3	Ft their CP & r provided $PT^2 > 0$
	Total		**9**	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$2\sin\theta\cos\theta$	B1	1	
(ii)	$2\cos^2\theta - 1$	B1	1	
(iii)	$\tan\theta(2\cos^2\theta)$	M1		Sub their "$\cos 2\theta$" expression
	$= 2\sin\theta\cos\theta = \sin 2\theta$	A1	2	**ag** be convinced
(b)	$\sin 2\theta = 2\sin^2 2\theta$	B1		
	$\sin 2\theta = 0$	M1		
	$\Rightarrow \theta = \frac{\pi}{2}$	A1		Accept 1.57 ... or 90° Ignore 0, π or any values outside interval
	$\sin 2\theta = \frac{1}{2}$	M1		$\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$
	$\Rightarrow \theta = \frac{\pi}{12}$	A1		accept 0.083π or better accept 15° or 75° if A1 not awarded for 90°
	$\Rightarrow \theta = \frac{5\pi}{12}$	A1	6	accept 0.417π or better (NOT 0.416π) All 3 must be correct and in terms of π and no extra solutions for final A1
	Total		**10**	
5(a)	$x = \frac{\ln 7}{\ln 3}\left(or \frac{\log_{10} 7}{\log_{10} 3}\right)$	M1		
	$= 1.77$	A1	2	condone more sf 1.77124....
(b)(i)		B1		$y = 3^x$ general shape
		B1	2	$y = 7 - x^2$ general shape
(ii)	2 roots	B1√	1	ft their graphs
(c)(i)	$x\ln 3 = \ln(7 - x^2) \Rightarrow x = \frac{\ln(7 - x^2)}{\ln 3}$	B1	1	**ag** be convinced
(ii)	$x_2 = 1.418$	B1		accept 1.42 or more SF 1.418284...
	$x_3 = 1.463$	B1	2	Must be 3 dp
	Total		**8**	

MBP4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$y^2 = 1 + \frac{12}{3x+2} + \frac{36}{(3x+2)^2}$	M1 A1	2	3 non-zero terms and 2 of them correct all correct
(b)(i)	$\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln(3x+2)$	M1 A1	2	$k\ln(3x+2)$ correct – condone missing +constant
(ii)	$\int \frac{1}{(3x+2)^2}\,dx = -\frac{1}{3}(3x+2)^{-1}$	M1 A1	2	$k(3x+2)^{-1}$ correct – condone missing +constant
(c)	$\pi\int_0^2 y^2 dx$	B1		
	$\left[x + 4\ln(3x+2) - 12(3x+2)^{-1}\right]$	M1 A1		Attempt to integrate "their" y^2 (2 terms) Correct unsimplified
	$(\pi)\left[2 + 4\ln(8/2) - 12(1/8 - 1/2)\right]$	m1		Evaluation of limits F(2) – F(0) (3 terms)
	$= 37.8$	A1	5	37.8410409...(condone more figures)
	Total		**11**	
7(a)(i)		B1 B1	2	General shape reflected in $y = x$ Asymptotes $y = \pm\frac{\pi}{6}$; (may be implied by numbers on y-axis); gradient > 0
(ii)	$y = \tan 3x \Rightarrow x = \frac{1}{3}\tan^{-1} y$	M1		Good attempt at x = ...
	$f^{-1}(x) = \frac{1}{3}\tan^{-1} x$	A1	2	
(b)(i)	$\frac{dx}{dy} = 3\sec^2 3y$	M1 A1	2	$k\sec^2$ ** correct
(ii)	$\frac{dy}{dx} = 1 \div their\ \frac{dx}{dy}$	M1		
	$= \frac{1}{3\sec^2 3y}$	A1		or $\frac{1}{3\sec^2(\pi/3)}$
	When $y = \frac{\pi}{9}$, $\frac{dy}{dx} = \frac{1}{12}$	A1	3	Accept 0.083 (or better) if all working shown
	Total		**9**	
	TOTAL		**60**	