GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP3)

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
-*x* EE		deduct *x* marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
SF		significant figure(s)
DP		decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working .... mark as in scheme

Incorrect answer without working .... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

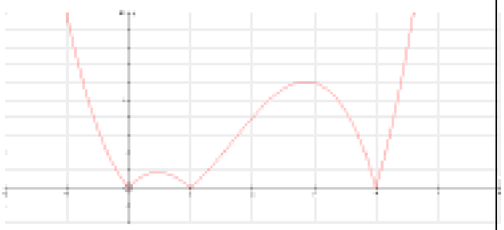
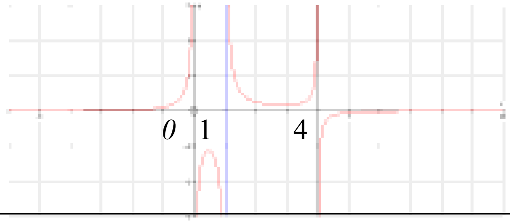
Mathematics and Statistics B Pure 3 MBP3 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$z = 3\sqrt{2}\left(\cos\frac{3\pi}{4} + \text{i}\ \sin\frac{3\pi}{4}\right) = -3 + 3\text{i}$	M1 A1	2	Give M1 if either a, b correct
(b)(i)	$w^2 = 1 - 3 - 2\,\text{i}\sqrt{3}$	M1		
	so that $w^2 + 2w = -4 \in \mathbb{R}$	A1	2	
(ii)	$\frac{4}{w} = \frac{4}{-1+\text{i}\sqrt{3}} \times \frac{-1-\text{i}\sqrt{3}}{-1-\text{i}\sqrt{3}}$	M1		
	$= -1 - \text{i}\sqrt{3}$	A1		
	$w - \frac{4}{w} = 2\text{i}\sqrt{3}$	A1	3	cao
	Total		**7**	
2	Multiplying by $(3-x)^2$	M1		
	$(3-x)\{3x+1-2(3-x)\} > 0$	m1		Collecting up on one side
	$5(x-1)(x-3) < 0$	B1√		$x = 1$, 3 identified ft
	$1 < x < 3$	A1	4	cao
	ALTERNATIVE 1:			
	For $x < 3$, $3x + 1 > 2(3 - x)$	M1		≥ 1 case correctly considered
	$\Rightarrow x > 1$	A1		
	For $x > 3$, $3x + 1 < 2(3 - x)$	M1		
	$\Rightarrow x < 1 \Rightarrow$ no solns.	A1	(4)	Must have a definite conclusion
	ALTERNATIVE 2:			
	Relevant graph drawn	M1 A1		Ignore irrelevant y-values
	Identifying correct intersections	B1√		ft if appropriate
	Correct range deduced	A1	(4)	
	Total		**4**	
3(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	M1		Must be evidence of correct matrix multn. method
	$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	A1		cao
(ii)	$\mathbf{M}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1√	3	ft
(b)	Rotation	M1		
	(anticlockwise) through ½π about 0	A1	2	"acw" may be implicit
(c)	$\mathbf{N} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	M1 A1	2	Any suitable method made clear
	Total		**7**	

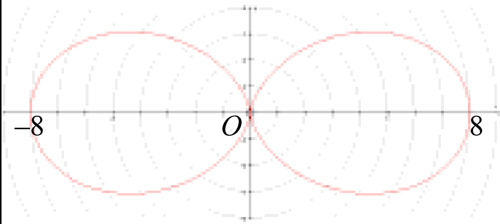
MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$\alpha + \beta = 2$, $\alpha\beta = ½$	B1 B1	2	
(ii)	$(\alpha + \beta)^2 - 2\alpha\beta = 3$	M1 A1√	2	ft (i)'s answers
(iii)	$\alpha^2 + \beta^2 + 6(\alpha + \beta) + 18 = 33$	M1 A1√	2	ft (i) and (ii)'s answers
(b)	New product of roots = 1 New sum of roots = $\dfrac{(\alpha+3)^2+(\beta+3)^2}{\alpha\beta+3(\alpha+\beta)+9}$ $= \dfrac{66}{31}$ New eqn. is $31y^2 - 66y + 31 = 0$	B1 M1 A1√ A1√	4	Form ready for substn. ft ft. Must have integer coefficients and be an equation (coefft. $y^2 \neq 1$)
	Total		**10**	
5(a)	$\ln y = \ln a + x \ln b$	B1	1	
(b)(i)	$\ln y$ 1.128 1.261 1.394 1.528 1.660 Points plotted on graph provided	B1 B1 B1	3	3 roots (to ≥ 3 s.f.) All roots to 3 d.p. (condone 1.66) Reasonably accurately
(ii)	"Good" straight line drawn	B1	1	
(c)(i)	From graph $x = 3.4 \Rightarrow \ln y = 1.44/5$ $\Rightarrow y = 4.24$ to 4.26	M1 A1	2	Including un-logging attempt awrt
(ii)	Method for finding gradient: $\ln b \approx \dfrac{0.67}{0.5} \approx 1.32\text{-}4$ $b = 3.7\text{-}3.9$ Reading off y-intercept: $\ln a \approx 0.99$ $a = 2.7$	 M1 A1 M1 A1	4	Sim. Eqns. Approach OK also For either/both M's awrt awrt
	Total		**11**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	 0 *1* *4*	M1 A1	 2	All above x-axis Good graph, with cusps. Ignore vertical scale.
(b)	$x = 0$, $x = 1$, $x = 4$, $y = 0$	B1 B1		Any 2 asymptotes stated All 4
		B1 B1 B1	 5	Region $0 < x < 1$ correct Region $1 < x < 4$ correct Regions $x < 0$, $x > 4$ correct
	Total		**7**	
7(a)	$a = 11$, $b = 9$, $c = 3$, $d = 1$	B1 B1	2	2 roots ; all 4 roots
(b)	$k = 9$	B1	1	
(c)	$3^{-1} = 5$	B1	1	
(d)	$x = 11$	B1	1	
(e)	$k^2 = 81 \equiv 11 \pmod{14} \equiv k + 2$	M1 A1	2	ft M only for k^2 (mod 14) correct
	Total		**7**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
8(a)(i)	Translation (// x-axis), vector $\begin{bmatrix}2\\0\end{bmatrix}$	M1 A1	2	B1 for equivalent correct description without "translation"
(ii)	$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$	M1		Backwards approach is fine
	$r^2(\cos^2\theta + \sin^2\theta) - 4r\cos\theta + 4 = 4$	A1		
	Use of $c^2 + s^2 = 1$	B1		
	$(r \neq 0) \Rightarrow r = 4\cos\theta$	A1	4	**ag**
(b)(i)	$r_{max} = 8$, $r_{min} = 0$	B1 B1	2	
(ii)		B1 B1 B1	3	Symmetry in $\theta = \frac{1}{2}\pi$ Symmetry in $\theta = 0$ All correct
(c)	Equating $8\cos^2\theta = 4\cos\theta$ and solving	M1		
	$\theta = \frac{1}{3}\pi$ and $r = 2$	A1 A1		
	2nd point $\theta = -\frac{1}{3}\pi$, $r = 2$	A1√	4	Or ft $2\pi - (1^{st}\ \theta)$, same r
	Total		**15**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
9(a)	For $n = 1$, LHS = RHS = 96	B1		True case $n = 1$
	Clear induction hypothesis somewhere	E1		
	Correct $(k+1)^{\text{th}}$ term used	B1		$4(k+2)(k+3)(k+4)$
	Some $(k+1)^{\text{th}}$ term added both sides	M1		
	$(k+2)(k+3)(k+4)\{k+1+4\}-24$ $=[(k+1)+1][(k+1)+2][(k+1)+3]\ldots$ $\ldots[(k+1)+4]-24$	m1		Factorising attempt
	Or explaining that formula true for $n = k \Rightarrow$ true also for $n = k + 1$	A1	6	Convincingly
(b)(i)	$\dfrac{r+3-(r+1)}{(r+1)(r+2)(r+3)} \equiv \dfrac{2}{(r+1)(r+2)(r+3)}$	B1	1	Shown
	$\sum \dfrac{2}{(r+1)(r+2)(r+3)} = \sum \dfrac{1}{(r+1)(r+2)} - \sum \dfrac{1}{(r+1)(r+2)}$	M1		Attempt at difference of two series
	$= \left\{\dfrac{1}{2.3}+\dfrac{1}{3.4}+\ldots+\dfrac{1}{(n+1)(n+2)}\right\} -$ $\left\{\dfrac{1}{3.4}+\ldots+\dfrac{1}{(n+1)(n+2)}+\dfrac{1}{(n+2)(n+3)}\right\}$	A1		Correct series identified
	$= \dfrac{1}{2.3} - \dfrac{1}{(n+2)(n+3)}$	m1		All terms except 1st and last cancelling; $A = \dfrac{1}{6}$
		A1	4	Give B1 if A deduced correctly
(iii)	$S = \dfrac{1}{6}$	B1$\surd$	1	ft their A
	Total		**12**	
	TOTAL		**80**	