General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP2**
(SPECIFICATION B)
Unit Pure 2

Tuesday 18 January 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A geometric series with positive terms is such that the fourth term is 64 times the seventh term.

(a) (i) Prove that the common ratio, r, of the series satisfies the equation

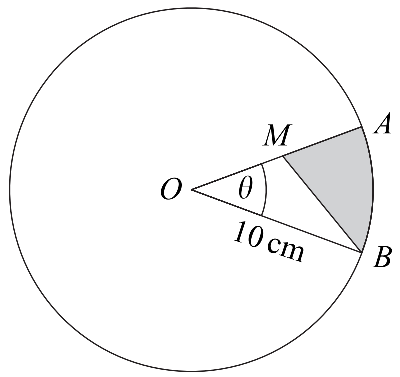
$$r^3 = \frac{1}{64}$$

(2 marks)

(ii) Hence find the value of r. *(1 mark)*

(b) Given that the first term, a, of the series is 12, find the sum to infinity of the series. *(2 marks)*

2



The diagram shows a sector OAB of a circle with centre O and radius 10 cm. The angle of the sector is θ radians. The point M is the midpoint of the radius OA. The region bounded by AM, MB and the arc AB is shaded.

(a) Find, in terms of θ, the area of the shaded region. *(4 marks)*

(b) Given that θ is small, use the small angle approximation for $\sin\theta$ to show that the area of the shaded region is approximately 25θ cm^2. *(2 marks)*

3 An antique is worth £V after t years, where

$$V = 300 + 150 \ln t, \quad \text{for } t \geqslant 1$$

(a) Find V when $t = 1$. *(1 mark)*

(b) Find t when $V = 600$. *(2 marks)*

(c) (i) Find $\frac{dV}{dt}$. *(1 mark)*

(ii) Find the rate of change of V when $t = 3$. *(2 marks)*

4 A polynomial is given by $p(x) = 6x^3 - 7x^2 - x + 2$.

(a) Find the value of $p\left(-\frac{1}{2}\right)$. *(1 mark)*

(b) Use the factor theorem to show that $(x - 1)$ is a factor of $p(x)$. *(2 marks)*

(c) Write $p(x)$ as a product of three linear factors. *(3 marks)*

(d) Hence find the values of θ, in radians, in the interval $-\pi < \theta < \pi$, for which

$$6\cos^3\theta - 7\cos^2\theta - \cos\theta + 2 = 0$$

(6 marks)

5 A curve has equation $y = e^x - 3x + 7$.

(a) (i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) The curve has a single stationary point at P. Show that the y-coordinate of P is $10 - \ln 27$. *(4 marks)*

(b) (i) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence determine whether P is a maximum or a minimum point. *(2 marks)*

(c) (i) Find $\int (e^x - 3x + 7)\,dx$. *(2 marks)*

(ii) Hence calculate the area of the region bounded by the curve $y = e^x - 3x + 7$, the x-axis and the lines $x = 0$ and $x = 2$. Give your answer in the form $e^a + b$, where a and b are integers to be found.

(You may assume that this region lies entirely above the x-axis.) *(3 marks)*

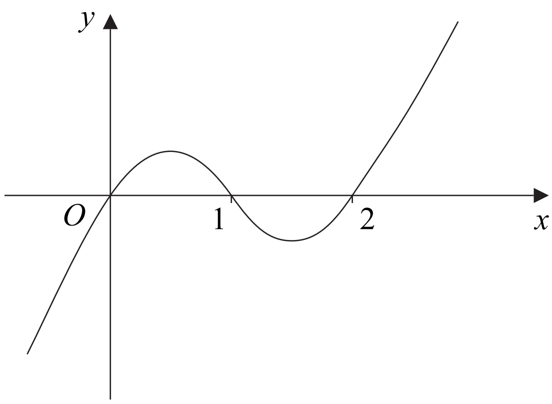
6 (a) Given that $\log_a x = m$ and $\log_a y = n$, find, in terms of m and n:

(i) $\log_a xy$; *(1 mark)*

(ii) $\log_a \left(\frac{x^2}{y}\right)$. *(2 marks)*

(b) Find the value of $\log_3 6$, giving your answer to three significant figures. *(2 marks)*

7 The diagram shows a sketch of the curve with equation $y = x(x-1)(x-2)$.



(a) Use the sketch to solve the inequality $x(x-1)(x-2) < 0$. *(2 marks)*

(b) (i) Expand the brackets $x(x-1)(x-2)$. *(2 marks)*

(ii) Find the x-coordinates of the two points on the curve at which the gradient of the curve is 11. *(5 marks)*

(c) (i) Sketch the curve with equation $y = |x(x-1)(x-2)|$. *(2 marks)*

(ii) The line $y = 6$ intersects the curve $y = |x(x-1)(x-2)|$ at the point (3, 6) and at the point P. Write down the coordinates of the point P. *(1 mark)*

END OF QUESTIONS