GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP2)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

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Question Number and Part	Solution	Marks	Total	Comments
1(a)(i)	4^{th} term = ar^3; 7^{th} term = ar^6			
	$\Rightarrow ar^3 = 64ar^6$. {Since $ar^3 \neq 0$}	M1		For use of ar^{n-1} (or ar^n)
	$\Rightarrow 64r^3 = 1 \Rightarrow r^3 = \frac{1}{64}$	A1	2	**ag** Be convinced
(ii)	$r = \frac{1}{4}$	B1	1	If unsimplified look for later evidence.
(b)	$S_\infty = \frac{a}{1-r}$	M1		
	$\Rightarrow S_\infty = \frac{12}{1-\frac{1}{4}} = 16$	A1√	2	ft on cand's r provided $\lvert r \rvert < 1$
	Total		**5**	
2(a)	Area of sector = $\frac{1}{2}r^2\theta$; [= $\frac{1}{2}10^2\theta$]	M1		
	Area of triangle OMB = $\frac{1}{2}\times 5\times 10\times \sin\theta$	M1		Use of $\frac{1}{2}ab\sin C$ oe
	Area of shaded region $= \frac{1}{2}10^2\theta - \frac{1}{2}\times 5\times 10\times \sin\theta$	m1		Dep on at least one M
	$= 50\theta - 25\sin\theta$	A1	4	
(b)	For small θ, $\sin\theta \approx \theta$	B1		
	Shaded area $\approx 50\theta - 25\theta = 25\theta$	B1	2	**ag** Be convinced
	Total		**6**	
3(a)	300	B1	1	
(b)	$600 = 300 + 150 \ln t \Rightarrow \ln t = 2$	M1		As far as $\ln t = k$
	$t = e^2$ [= 7.38(9..)]	A1	2	Condone $t = 7.4$
(c)(i)	$\frac{dV}{dt} = \frac{150}{t}$	B1	1	
(ii)	Rate of change when $t = 3$ is $V'(3)$	M1		Recognises need for $V'(3)$
	$V'(3) = \frac{150}{3} = 50$	A1√	2	ft on $V'(t) = (p) + \frac{q}{t}$, $q > 0$
	Total		**6**	

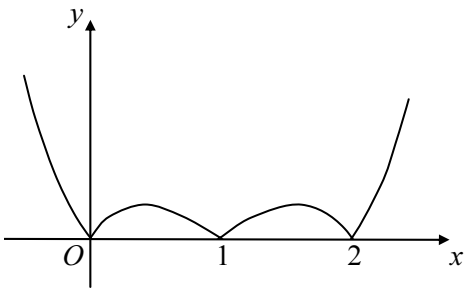
MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	$p\left(-\frac{1}{2}\right) = -\frac{6}{8} - \frac{7}{4} + \frac{1}{2} + 2 = 0$	B1	1	
(b)	$p(1) = 6 - 7 - 1 + 2$	M1		Use of p(1). Must use F.Thm.
	$= 0 \Rightarrow (x - 1)$ is a factor of $p(x)$	A1	2	**ag.** Must have a statement
(c)	From (a), $(2x + 1)$ is a factor	B1		Accept seen.
	$(x - 1)(2x + 1)[3x \ldots . - 2]$	M1		Valid attempt at 3rd factor (coeff of x^3 or const correct) or p(2/3) attempted
	$p(x) \equiv (x - 1)(2x + 1)(3x - 2)$	A1	3	
(d)	$x \to \cos\theta \Rightarrow$			
	$(\cos\theta - 1)(2\cos\theta + 1)(3\cos\theta - 2) = 0$	M1		using $x = \cos\theta$. PI
	$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$	B1		
	$\cos\theta = -\frac{1}{2}; \Rightarrow \theta = \frac{2\pi}{3}$ (= 2.09…)	A1		
	$\theta = -\frac{2\pi}{3}$	A1√		ft on $-$"$\frac{2\pi}{3}$" provided correct quadrants used for cand's factor
	$\cos\theta = \frac{2}{3}; \Rightarrow \theta = 0.84(1..)$	A1√		ft on cand's 3rd factor only if $\lvert\cos\theta\rvert \leq 1$
	$\theta = -0.84(1..)$	A1√	6	Deduct max of 1 from accuracy marks if answers in degrees. Ignore answers outside $-\pi < \theta < \pi$
	Total		**12**	

MBP2(cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)(i)	$\frac{dy}{dx} = e^x - 3$	B2,1,0	2	
(ii)	At st. pt $e^x - 3 = 0$ $\Rightarrow e^x = 3$ $\Rightarrow x = \ln 3$ (= 1.098..) $y = 3 - 3\ln 3 + 7 = 10 - \ln 27$	M1 A1√ m1 A1	4	Puts $y'(x) = 0$ ft on one slip in (a) to $e^x = k$ e^x to x via ln **ag** Be convinced. No decimals used
(b)(i)	$\frac{d^2y}{dx^2} = e^x$	B1	1	
(ii)	$e^x > 0$; $y''(x) > 0$; $y''(\ln 3) > 0$ $\Rightarrow$ st pt is a minimum	B1√ B1√	2	ft on (b)(i) (any one of the three oe) ft on candidate's sign of y'' provided no 'dubious' statement.
(c)(i)	$\int e^x - 3x + 7dx = e^x - \frac{3x^2}{2} + 7x \{+ c\}$	B2,1,0	2	
(ii)	Area $= \left[e^x - \frac{3x^2}{2} + 7x \right]_0^2$ $= (e^2 - 6 + 14) - (e^0)$ $= e^2 + 7$	B1√ M1 A1	3	PI F(2) – F(0)
	Total		**14**	
6(a)(i)	$\log_a xy = m + n$	B1	1	
(ii)	$\log_a \left(\frac{x^2}{y}\right) = \log_a x^2 - \log_a y$ $= 2\log_a x - \log_a y$ $= 2m - n$	M1 A1	2	Use of one law of logs. PI
(b)	$\log_3 6 = \frac{\ln 6}{\ln 3}$ oe $= 1.6309\ldots = 1.63$ to 3sf	M1 A1	2	
	Total		**5**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	$x < 0$; $1 < x < 2$	B1 B1	2	Deduct max of 1 from any B marks earned if ≤ used
(b)(i)	$x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$	B2,1,0	2	–1 each indep.error
(ii)	$y'(x) = 3x^2 - 6x + 2$	M1		
	$3x^2 - 6x + 2 = 11$	m1		Puts $y'(x) = 11$
	$3(x^2 - 2x - 3) = 0$	A1		Factor 3 not needed
	$(x - 3)(x + 1) = 0$	m1		Solve or factorise. Dep on both previous M and m.
	$x = 3$, $x = -1$	A1	5	Need both values.
(c)(i)		B2,1,0	2	B1 for graph correct for either $0 < x < 2$ or for both $x < 0$ and $x > 2$
(ii)	$P(-1, 6)$	B1	1	
	Total		**12**	
	TOTAL		**60**	