General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) MBM3
Unit Mechanics 3

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A car is travelling along a straight horizontal road. When the car is at a distance of 48 metres from a set of traffic lights, it is travelling at $20\,\mathrm{m\,s^{-1}}$. The car slows down uniformly so that it has a speed of $4\,\mathrm{m\,s^{-1}}$ as it passes the traffic lights.

(a) Show that the acceleration of the car is $-4\,\mathrm{m\,s^{-2}}$. *(2 marks)*

(b) How far past the traffic lights does the car travel before it stops? *(3 marks)*

(c) Calculate the total time that it takes the car to stop. *(2 marks)*

(d) The car has mass 1100 kg. As the car slows down, it experiences a resistance force that has a constant magnitude of 200 newtons, and a force due to the brakes of magnitude F newtons.

Find F. *(3 marks)*

2 A particle, of mass 2 kg, is set into motion up a rough slope inclined at 40° to the horizontal. The coefficient of friction between the particle and the slope is 0.3 . Assume that there is no air resistance acting on the particle.

(a) Find the magnitude of the friction force that acts on the particle. *(3 marks)*

(b) Find the acceleration of the particle as it moves up the slope. *(3 marks)*

3 A particle moves so that, at time t seconds, its position vector, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = 4\mathrm{e}^{-t}\mathbf{i} + (6t + 3\mathrm{e}^{-t})\mathbf{j}$$

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.

(a) Show that the velocity of the particle is $(-4\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ when $t = 0$. *(3 marks)*

(b) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle when $t = 0$. *(2 marks)*

(d) Describe what happens to the velocity of the particle as t becomes large. *(2 marks)*

4 A small sphere, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is fixed to a smooth horizontal surface at the point O. The string has modulus of elasticity 40 N and natural length 2 metres.

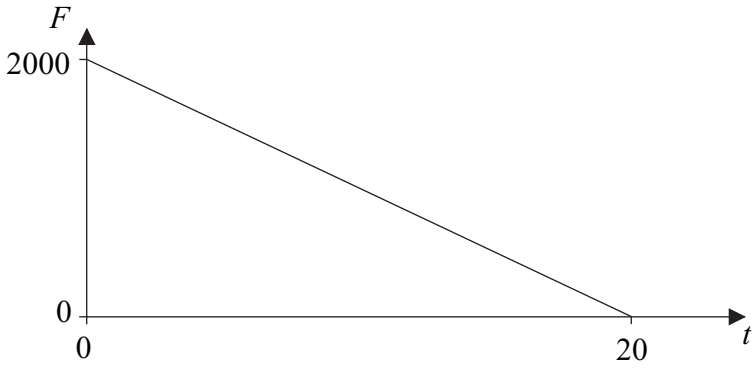
The sphere is released from rest at the point P, which is on the horizontal surface 5 metres from O.

(a) Show that the elastic potential energy of the string is 90 J when the sphere is released. *(2 marks)*

(b) Show that the speed of the sphere is $6\,\mathrm{m\,s^{-1}}$ when the string becomes slack. *(3 marks)*

(c) Find the speed of the sphere when it is 3 metres from O. *(5 marks)*

5 A car, of mass 1000 kg, is travelling along a straight horizontal road. Its brakes are applied for a period of 20 seconds. The graph below shows how the magnitude of the braking force, F newtons, varies with time, t seconds, for $0 \leqslant t \leqslant 20$.



(a) The magnitude of the braking force, at time t, can be expressed as

$$F = P - Qt$$

where P and Q are constants.

Use the graph to find the values of P and Q. *(2 marks)*

(b) The braking force is the only horizontal force that acts on the car. Show that the acceleration of the car, at time t, is given by

$$a = \frac{t}{10} - 2$$ *(2 marks)*

(c) The car comes to rest when $t = 20$. Find an expression for the velocity of the car at time t. *(4 marks)*

(d) Find the distance that the car travels in the 20 seconds. *(5 marks)*

Turn over ▶

6 The mass of a car is 1500 kg. The car travels up a slope inclined at 5° to the horizontal. When the car moves at a speed of $v\,\mathrm{m\,s^{-1}}$, it is subject to a resistance force of magnitude $30v$ newtons. Model the car as a particle.

(a) Draw a diagram to show the forces acting on the car. *(1 mark)*

(b) Show that the power output of the car when it travels up the slope at a constant speed of $10\,\mathrm{m\,s^{-1}}$ is 15 800 W, correct to 3 significant figures. *(4 marks)*

(c) The car has a maximum power output of 35 000 W. Find the maximum possible speed of the car as it travels, from rest, up the slope. *(6 marks)*

7 A particle moves with constant acceleration $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 0$, the particle is at the origin and has velocity $(3\mathbf{i} - 10\mathbf{j})\,\mathrm{m\,s^{-1}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Show that the velocity of the particle at time t seconds is

$$(3 + 4t)\mathbf{i} + (2t - 10)\mathbf{j}$$ *(2 marks)*

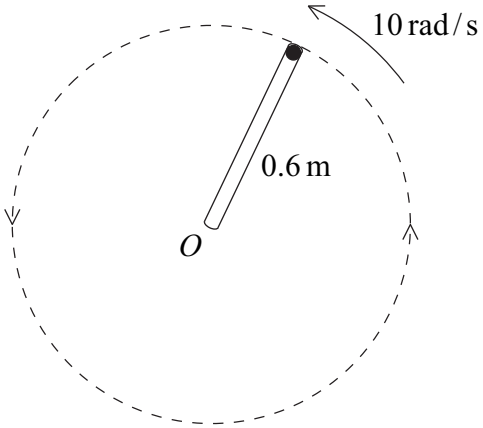
(b) Find the time when the particle is travelling due east. *(2 marks)*

(c) (i) Find the position vector of the particle when $t = 10$. *(3 marks)*

(ii) Find the velocity of the particle when $t = 10$. *(1 mark)*

(d) The particle stops accelerating when $t = 10$ and then moves with constant velocity. Find the distance of the particle from the origin after it has been travelling with this constant velocity for a further 10 seconds. *(6 marks)*

8 The diagram shows a smooth tube, sealed at each end, which rotates about one end. There is a small sphere inside the tube. The tube rotates in a vertical plane, so that the sphere remains at the end of the tube and describes a vertical circle of radius 0.6 metres. The mass of the sphere is 0.05 kg and the tube rotates with a constant angular speed of 10 radians per second.



(a) Calculate the magnitude of the acceleration of the sphere. *(2 marks)*

(b) Find the magnitude of the horizontal reaction force exerted by the end of the tube on the sphere when the tube is horizontal. *(2 marks)*

(c) Find the magnitude of the vertical reaction force exerted by the end of the tube on the sphere when the sphere is at its lowest point. *(3 marks)*

END OF QUESTIONS

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