General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 2

MBM2

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A particle moves so that, at time t seconds, its position vector, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = 4e^{-t}\mathbf{i} + (6t + 3e^{-t})\mathbf{j}$$

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.

(a) Show that the velocity of the particle is $(-4\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ when $t = 0$. *(3 marks)*

(b) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle when $t = 0$. *(2 marks)*

(d) Describe what happens to the velocity of the particle as t becomes large. *(2 marks)*

2 A small sphere, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is fixed to a smooth horizontal surface at the point O. The string has modulus of elasticity 40 N and natural length 2 metres.

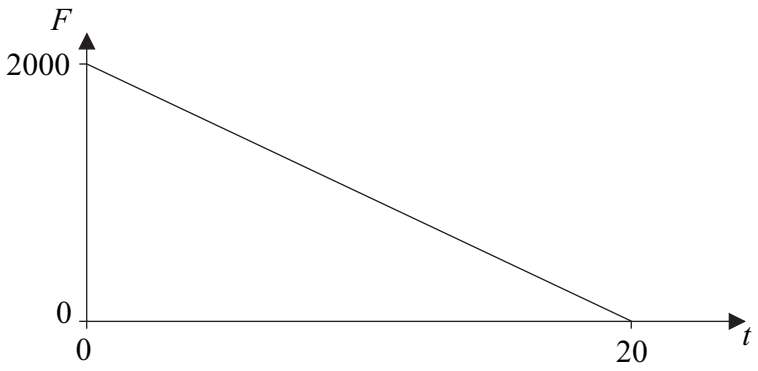
The sphere is released from rest at the point P, which is on the horizontal surface 5 metres from O.

(a) Show that the elastic potential energy of the string is 90 J when the sphere is released. *(2 marks)*

(b) Show that the speed of the sphere is $6\,\mathrm{m\,s^{-1}}$ when the string becomes slack. *(3 marks)*

(c) Find the speed of the sphere when it is 3 metres from O. *(5 marks)*

3 A car, of mass 1000 kg, is travelling along a straight horizontal road. Its brakes are applied for a period of 20 seconds. The graph below shows how the magnitude of the braking force, F newtons, varies with time, t seconds, for $0 \leqslant t \leqslant 20$.



(a) The magnitude of the braking force, at time t, can be expressed as

$$F = P - Qt$$

where P and Q are constants.

Use the graph to find the values of P and Q. *(2 marks)*

(b) The braking force is the only horizontal force that acts on the car. Show that the acceleration of the car, at time t, is given by

$$a = \frac{t}{10} - 2$$ *(2 marks)*

(c) The car comes to rest when $t = 20$. Find an expression for the velocity of the car at time t. *(4 marks)*

(d) Find the distance that the car travels in the 20 seconds. *(5 marks)*

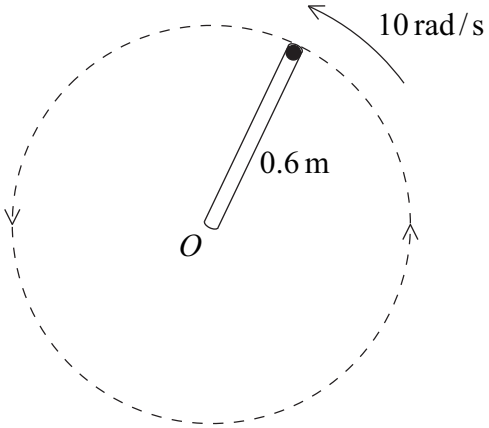
4 The mass of a car is 1500 kg. The car travels up a slope inclined at 5° to the horizontal. When the car moves at a speed of $v\,\mathrm{m\,s^{-1}}$, it is subject to a resistance force of magnitude $30v$ newtons. Model the car as a particle.

(a) Draw a diagram to show the forces acting on the car. *(1 mark)*

(b) Show that the power output of the car when it travels up the slope at a constant speed of $10\,\mathrm{m\,s^{-1}}$ is 15 800 W, correct to 3 significant figures. *(4 marks)*

(c) The car has a maximum power output of 35 000 W. Find the maximum possible speed of the car as it travels, from rest, up the slope. *(6 marks)*

5 The diagram shows a smooth tube, sealed at each end, which rotates about one end. There is a small sphere inside the tube. The tube rotates in a vertical plane, so that the sphere remains at the end of the tube and describes a vertical circle of radius 0.6 metres. The mass of the sphere is 0.05 kg and the tube rotates with a constant angular speed of 10 radians per second.

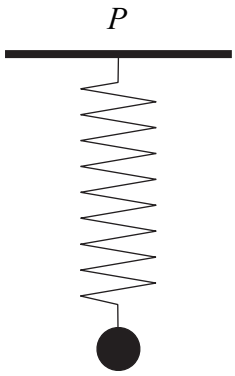


(a) Calculate the magnitude of the acceleration of the sphere. *(2 marks)*

(b) Find the magnitude of the horizontal reaction force exerted by the end of the tube on the sphere when the tube is horizontal. *(2 marks)*

(c) Find the magnitude of the vertical reaction force exerted by the end of the tube on the sphere when the sphere is at its lowest point. *(3 marks)*

6 A sphere of mass 2 kg is fixed to one end of a spring. The other end of the spring is attached to the fixed point P, as shown in the diagram. The natural length of the spring is 0.5 metres.



(a) When the system is in equilibrium, the length of the spring is 0.7 metres.

Find the modulus of elasticity of the spring. *(3 marks)*

(b) The sphere is pulled down below its equilibrium position and released from rest. At time t seconds, the displacement of the sphere from its equilibrium position is x metres.

(i) Show that

$$\frac{d^2x}{dt^2} = -49x$$ *(5 marks)*

(ii) Find the value of t when the sphere returns to its equilibrium position for the first time. *(4 marks)*

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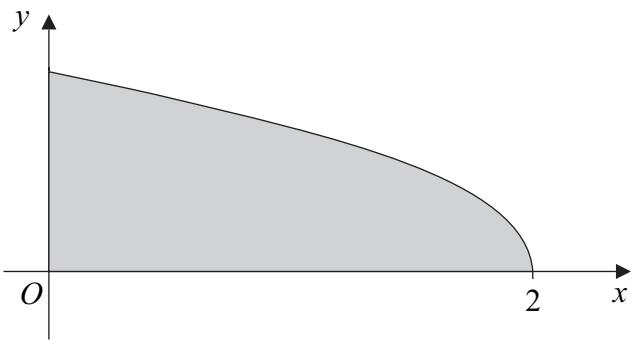
7 The region bounded by the x-axis, the y-axis and the curve with equation $y = \sqrt{2 - x}$ is shown in the diagram below.



The region is rotated through 360° around the x-axis to form a uniform solid.

(a) Show that the volume of the solid is 2π. *(3 marks)*

(b) Find the distance of the centre of mass of the solid from the y-axis. *(4 marks)*

(c) The solid is suspended from a point on the edge of its circular face and hangs in equilibrium. Find the angle between the circular face and the vertical. *(4 marks)*

8 A boat, of mass m, is travelling at a constant speed U when its motor fails. As it slows down, it moves in a straight line. While it is slowing down, it experiences a resistance force of magnitude kv^2, where k is a constant and v is the speed of the boat.

Find the speed of the boat when it has travelled x metres. *(7 marks)*

END OF QUESTIONS

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