General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBD2**
(SPECIFICATION B)
Unit Discrete 2

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Question 2 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The sequence $p_1, p_2, p_3, \ldots$ satisfies the recurrence relation

$$p_1 = 0, \qquad p_n = 2p_{n-1} + 1, \quad n > 1$$

(a) Solve the recurrence relation to derive an expression for p_n in terms of n. Simplify your answer as far as possible. *(4 marks)*

(b) Calculate the values of p_4 and p_5. *(1 mark)*

(c) In fact p_n equals the number of ways of splitting a set of n items into two separate parts. So, for example, $p_3 = 3$ since the set $\{a,b,c\}$ can be split into two parts in three ways, namely

$$\{a\} \text{ and } \{b,c\} \qquad \{a,b\} \text{ and } \{c\} \qquad \{a,c\} \text{ and } \{b\}$$

List the p_4 different ways of splitting the set $\{a,b,c,d\}$ into two parts. *(3 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in answering part (a)(ii) of this question.]

The towns *P*, *Q*, *R*, *S*, *T*, *U*, *V* and *W* are all linked to each other by direct roads whose lengths, in miles, are shown in the following table:

	P	***Q***	***R***	***S***	***T***	***U***	***V***	***W***
P	—	5	4	3	4	5	4	6
Q	5	—	7	5	3	5	8	8
R	4	7	—	6	8	6	3	5
S	3	5	6	—	4	6	9	6
T	4	3	8	4	—	7	8	6
U	5	5	6	6	7	—	8	3
V	4	8	3	9	8	8	—	8
W	6	8	5	6	6	3	8	—

(a) An estate agent wants to start at the town *P* and visit each of the other towns once before returning to *P*.

(i) Use the nearest neighbour algorithm to find one possible route for the estate agent and state its total length. *(4 marks)*

(ii) On **Figure 1**, ignore the town *P* and find the length of a minimum connector of the towns *Q*, *R*, *S*, *T*, *U*, *V* and *W*. *(4 marks)*

(iii) Deduce a lower bound for the estate agent's route and comment on its significance to your answer to part (a)(i). *(3 marks)*

(b) The estate agent would then like to look at the houses on each of the roads joining the towns. The estate agent starts at *P*, covers each of the roads at least once, and finishes at *P*.

(i) Explain why it will be necessary to cover at least four of the roads more than once. *(2 marks)*

(ii) Given that the total length of all the roads is 160 miles, explain how you know that all the roads can be covered with a round trip of 172 miles. *(3 marks)*

3 Oxfield College gives each of its students a 9-digit registration number

$$x_1x_2x_3x_4x_5x_6x_7x_8x_9$$

where the four digits $x_1x_2x_3x_4$ show the year in which the student started and the final digit x_9 is chosen so that

$$x_1 + 3x_2 + x_3 + 3x_4 + x_5 + 3x_6 + x_7 + 3x_8 + x_9$$

is divisible by 10.

(a) Explain the purpose of the last digit. *(1 mark)*

(b) Verify that 200432298 is a correct registration number. *(1 mark)*

(c) A student who started in 2004 writes his registration number as 200432299 but he has written one of the digits incorrectly. Find the five possible correct registration numbers. *(4 marks)*

(d) Another student incorrectly writes her registration number as 200432296 by writing two adjacent digits in the wrong order. What is her correct number? *(2 marks)*

(e) If each student is to have a different registration number, what is the maximum number of students that the college can accept in any one year? *(2 marks)*

(f) Show that in any correct registration number the sum of the nine digits is even. *(2 marks)*

4 A linear programming problem consists of maximising an objective function P in three variables x, y and z. The simplex method is used to solve the problem and several iterations of the method lead to the following tableau:

P	x	y	z	s	t	u	v	
1	−2	0	0	2	0	0	1	60
0	−2	0	1	1	0	0	−1	15
0	1	0	0	2	1	0	2	25
0	2	0	0	−1	0	1	1	40
0	1	1	0	0	0	0	4	30

(a) What is the name given to the variables s, t, u and v? *(1 mark)*

(b) Other than $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, how many inequalities are involved in the problem? *(1 mark)*

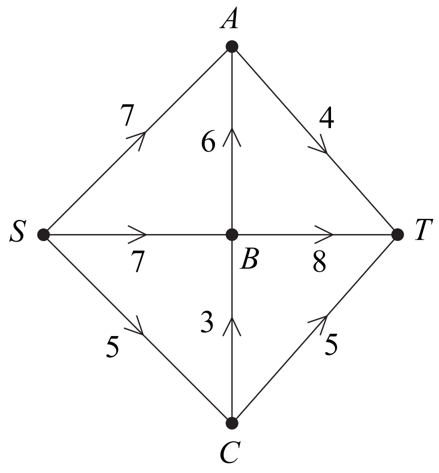
(c) Apply one further iteration of the simplex method to the above tableau. *(5 marks)*

(d) Explain why your new tableau solves the original problem. State the maximum value of P and the values of x, y and z for which that maximum is reached. *(3 marks)*

(e) State the values of s, t, u and v at the optimal point. How many of the inequalities from the original problem still have some slack? *(2 marks)*

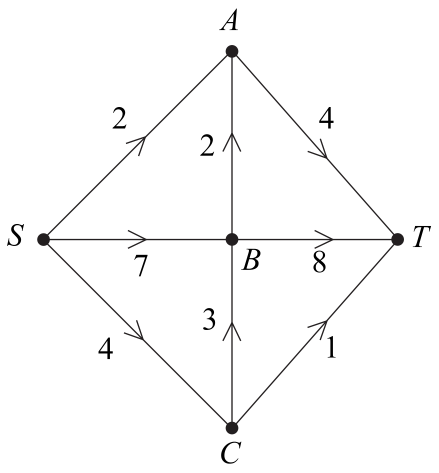
TURN OVER FOR THE NEXT QUESTION

5 The diagram shows the capacities of the arcs of a network.



(a) Find a cut of 16 in the network above. *(2 marks)*

(b) The next diagram shows a flow of 13 from S to T in the same network.



(i) Find a flow-augmenting path from S to T which uses just one other vertex and which raises the given flow of 13 to a flow of 14. *(2 marks)*

(ii) Find a further flow-augmenting path from S to T which raises the flow from 14 to 16. *(2 marks)*

(c) How do you know that the flow of 16 is the maximum possible in this network? *(1 mark)*

(d) The capacity of one arc of the network is to be increased in order to enable a flow of 19 from S to T. State which arc should be chosen and give a reason. *(3 marks)*

6 (a) Find the general solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 0$$ *(4 marks)*

(b) Given that k is a constant and $u_n = kn$ satisfies the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16$$

find the value of k. *(3 marks)*

(c) Write down the general solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16$$ *(1 mark)*

(d) Given that $u_0 = u_1 = 1$, find the solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16$$ *(4 marks)*

7 A linear binary code has parity check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

So, for example, 0001111 and 0011000 are two of the codewords.

(a) Calculate the other six codewords. *(4 marks)*

(b) Find the Hamming distance of the code and state the number of errors in a word which the code can detect. *(2 marks)*

(c) Given that the message

11001000110111

contains at most one error in each codeword, use the parity check matrix to find the two possible correct messages. *(4 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

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Insert for use in Question 2.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

	P	***Q***	***R***	***S***	***T***	***U***	***V***	***W***
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Q	5	—	7	5	3	5	8	8
R	4	7	—	6	8	6	3	5
S	3	5	6	—	4	6	9	6
T	4	3	8	4	—	7	8	6
U	5	5	6	6	7	—	8	3
V	4	8	3	9	8	8	—	8
W	6	8	5	6	6	3	8	—

Figure 1

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