GCE 2004
November Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
MBP2

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from:

Publications Department, Aldon House, 39, Heald Grove, Rusholme, Manchester, M14 4NA
Tel: 0161 953 1170

or

download from the AQA website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales 3644723 and a registered charity number 1073334. Registered address AQA, Devas Street, Manchester. M15 6EX. *Dr Michael Cresswell Director General*

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√or ft		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
PI		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC –*x*	deducted *x* marks for mis-copy
MR –*x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	gave benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

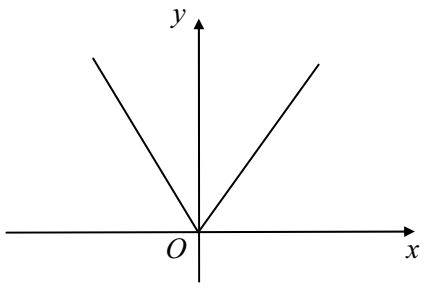
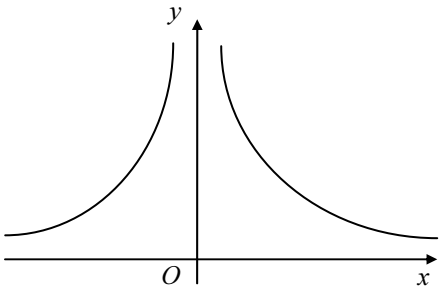
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Mathematics and Statistics B Pure 2 MBP2 November 2004

Question Number and Part	Solution	Marks	Total	Comments
1(a)	Arc length = $r\theta$	M1		
	$4 = 10\theta \Rightarrow \theta = 0.4$	A1	2	
(b)	Area of sector = $\frac{1}{2}r^2\theta$	M1		
	$= 50\ \theta = 20\ \text{cm}^2$	A1√	2	ft on candidate's θ Condone missing/wrong units
	Total		**4**	
2(a)(i)	e.g. $r = \frac{-576}{720} = -0.8$	B1	1	**ag** Be convinced
(ii)	When $r = -0.8$, $-1 < r < 1$ so series is convergent	E1	1	oe
(b)	n th term = $a\,r^{n-1}$	M1		condone n th term = ar^n
	$= 720(-0.8)^{n-1}$	A1	2	
(c)	$S_{15} = \frac{a\left(1-r^{15}\right)}{1-r}$	M1		
	$= \frac{720\left(1-(-0.8)^{15}\right)}{1-(-0.8)} = 414(.07)...$	A1	2	**ag** Need to see some evaluation or a more accurate answer
(d)	$\frac{a}{1-r} =$	M1		
	$\frac{720}{1-(-0.8)} = 400$	A1	2	
	Total		**8**	
3(a)	p(4) = 64 – 32 – 44 + 12	M1		p(4) attempted
	p(4) = 0 ⇒ $(x – 4)$ is a factor of p(x)	A1	2	**ag** Must have conclusion or equivalent earlier statement
(b)	$(x-4)\left[x^2 \ldots\ldots - 3\right]$	M1		coeff of x^3 or const correct or p(1) or p(–3) considered
	$(x-4)\left[x^2 + 2x - 3\right]$	A1		
	$(x – 4)\,[x – 1]\,[x + 3]$	m1		valid method to 3rd factor
	p(x) ≡ $(x – 4)\,(x – 1)(x + 3)$	A1	4	
(c)	$x \to y^2$			
	$(y^2 – 4)\,(y^2 – 1)(y^2 + 3) = 0$	M1		using $x = y^2$
	$y^2 = 4; \Rightarrow y = \pm 2$ $y^2 = 1; \Rightarrow y = \pm 1$ $y^2 = –3; \Rightarrow$ no solution	A2,1√	3	ft on (b) provided equivalent demands. A1ft for any three of five 'correct'. Accept ignoring negative value of y^2 without statement
	Total		**9**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4	$\sin\left(x+\frac{\pi}{3}\right)=-0.3$			
	$\sin^{-1}(0.3)=0.304\{69\ldots\}$	M1		Taking $\sin^{-1}(0.3)$; award if either 0.304... or – 0.304.. or 17.4{5..} or – 17.4{5..} seen
	$\Rightarrow \{X\} = \pi +$ "0.304{69…}"	m1		Angle in 3rd quadrant. Accept degrees; condone mix.
	or $\{X\} = 2\pi -$ "0.304{69…}"	m1		Angle in 4th quadrant. Accept degrees; condone mix.
	$x+\frac{\pi}{3}$ used for X	m1		Dep on M and at least one of the two m's
	$x = 2.39908\ldots = 2.40$	A1		Accept awrt in both answers. Deduct a max of 1 mark from any A marks if final answer(s) are in degrees.{137.457…; 282.54…} Accept 0.764π and 1.57π. (Both 2.39 & 4.94 can score A1) **NB** eg M1m1m0m1A1A0 is possible
	or $x = 4.931295\ldots = 4.93$	A1	6	
	Total		**6**	
5(a)		M1		Single V-shaped graph
		A1	2	Vertex at origin, and 'roughly' symmetrical
(b)		B2,1	2	B1 each branch
(c)(i)	$\frac{1}{\frac{1}{4}}-4=4-4=0$	B1	1	convincing verification
(ii)	$\left(\frac{1}{2},4\right)$ and $\left(-\frac{1}{2},4\right)$	B2,1	2	B1 for two of the four coordinates correct
	Total		**7**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)(i)	$f'(x) = 4e^{4x} + x^{-2}$	B1 M1 A1	3	For x^{-2} oe For $k\,e^{4x}$, $k \neq 0$ For $4\,e^{4x}$
(ii)	$e^{4x} > 0$ and $x^{-2} > 0$ {for $x > 0$} so $f'(x) > 0 \Rightarrow f$ is an increasing fn.	M1 A1	2	Award max. of M1A0 if insufficient detail
(b)	$\int\left(e^{4x} - \frac{1}{x}\right)dx = \frac{1}{4}e^{4x} - \ln x \ \{+c\}$	M1 A1		One term correct Both terms correct
	$\int_1^2\left(e^{4x} - \frac{1}{x}\right)dx = \left[\frac{1}{4}e^{4x} - \ln x\right]_1^2$			
	$= \left(\frac{1}{4}e^8 - \ln 2\right) - \left(\frac{1}{4}e^4 - \ln 1\right)$	M1		F(2) – F(1)
	$= \frac{e^4(e^4 - 1)}{4} - \ln 2$	A1	4	**ag** (must be exact throughout)
(c)	$e^{4x} - \frac{1}{x} = 7 - \frac{1}{x} \Rightarrow e^{4x} = 7$	M1		To $e^{ax} = b$ stage
	$\Rightarrow 4x = \ln 7$	m1		exponential to ln
	$\Rightarrow x = \frac{1}{4}\ln 7$	A1	3	Accept any equivalent **exact** form
	Total		**12**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)(i)	$\frac{dy}{dx} = \frac{2}{x} - 4$	M1 A1	2	Clear differentiation
(ii)	When $x = 2$, $\frac{dy}{dx} = \frac{2}{2} - 4 = -3$	A1√	1	Only ft if no log term
(b)	At st. pt., $y'(x) = 0 \Rightarrow \frac{2}{x} - 4 = 0$	M1		
	$\Rightarrow \quad x = \frac{1}{2}$	A1	2	**ag** Be convinced; cso
(c)(i)	$\frac{d^2y}{dx^2} = -\frac{2}{x^2}$	B1√	1	
(ii)	$x^2 > 0 \Rightarrow y''(x) < 0$ {alt. $y''\left(\frac{1}{2}\right) < 0$}	E1√		ft on non-constant (c)(i)
	$\Rightarrow P$ is a maximum	E1√	2	ft on candidate's sign of y''
(d)(i)	$4 = \frac{2}{x} - 4$	M1		
	$\Rightarrow \frac{2}{x} = 8 \Rightarrow x = \frac{1}{4}$	A1	2	**ag** Be convinced
(ii)	$Q\left(\frac{1}{4}, 2\ln\frac{1}{4} - 1\right)$; $P\left(\frac{1}{2}, 2\ln\frac{1}{2} - 2\right)$	M1		Finding y-coordinates; ln's involved or correct numerical values
	Grad of $PQ = \frac{\left(2\ln\frac{1}{2} - 2\right) - \left(2\ln\frac{1}{4} - 1\right)}{\frac{1}{2} - \frac{1}{4}}$	m1		Finding gradient
	$= 8\ln 2 - 4$	m1 A1	4	Using log law to reach $\ln k$ Must be in given form Accept $a = 8$, $b = -4$
	Total		**14**	
	TOTAL		**60**	