General Certificate of Education
November 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP1**
(SPECIFICATION B)
Unit Pure 1

Tuesday 2 November 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that $x^2 + 6x + 4 \equiv (x + a)^2 + b$, find the values of the constants a and b. *(2 marks)*

(b) Hence, or otherwise, find the exact solutions of the equation $x^2 + 6x + 4 = 0$. *(2 marks)*

2 The line PQ has equation $5x + 3y = 10$ and P is the point $(8, -10)$.

(a) (i) Find the gradient of the line PQ. *(1 mark)*

(ii) Find an equation of the line through P that is perpendicular to the line PQ. *(2 marks)*

(b) The line QR has equation $y = x - 6$. Calculate the coordinates of Q. *(3 marks)*

(c) Find the coordinates of the point S such that $(6, -4)$ is the midpoint of PS. *(2 marks)*

3 A curve has equation $y = x^3 - 3x^2 - 9x - 8$ and passes through the point $P(3, -35)$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the curve has a stationary point at P and find the coordinates of the other stationary point of the curve. *(4 marks)*

(c) State, with a reason, whether P is a maximum or minimum point. *(2 marks)*

(d) Show that the curve crosses the x-axis at a point where the x-coordinate lies between 5.0 and 5.1. *(2 marks)*

4 An arithmetic series has first term 5 and common difference 6.

(a) The sum of the first n terms of the series is S_n. Show that $S_n = 3n^2 + 2n$. *(3 marks)*

(b) Given that $S_n > 2640$:

(i) show that $(n + 30)(3n - 88) > 0$; *(1 mark)*

(ii) find the least possible value of the positive integer n. *(2 marks)*

5 (a) Solve the equation

$$\tan 3x = -1$$

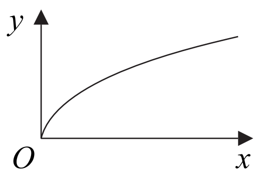
giving all solutions in the interval $-90° < x < 90°$. *(5 marks)*

(b) Describe fully the geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan 3x$. *(2 marks)*

6 The function f has domain $0 \leqslant x \leqslant 9$ and is defined by $f(x) = \sqrt{x} - 2$.

(a) (i) Find $f(0)$ and $f(9)$. *(1 mark)*

(ii) The graph of $y = \sqrt{x}$ for $x \geqslant 0$ is sketched below.



Hence sketch the graph of $y = f(x)$, stating clearly the values of the intercepts on the coordinate axes. *(3 marks)*

(b) Find the range of f. *(2 marks)*

(c) The inverse of f is f^{-1}.

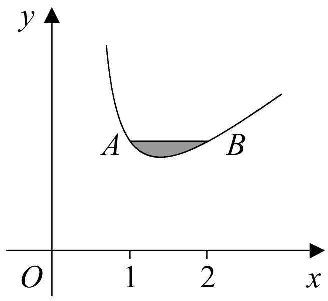
(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) State the domain of f^{-1}. *(2 marks)*

(iii) Sketch the graph of $y = f^{-1}(x)$. *(2 marks)*

Turn over ▶

7 The curve with equation $y = 14x + \dfrac{16}{x^3}$ is sketched below for $x > 0$.



The points A and B lie on the curve and have x-coordinates equal to 1 and 2 respectively.

(a) Find the y-coordinates of A and B and hence show that AB is parallel to the x-axis. *(2 marks)*

(b) (i) Find $\displaystyle\int \left(14x + \frac{16}{x^3}\right) \mathrm{d}x$. *(3 marks)*

(ii) Hence calculate the area of the region bounded by the curve and the line AB, shaded in the diagram. *(4 marks)*

(c) The function f is defined for all non-zero values of x by

$$\mathrm{f}(x) = 14x + \frac{16}{x^3}$$

Prove that f is an odd function. *(2 marks)*

END OF QUESTIONS