

General Certificate of Education
November 2004
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Discrete 1**

MBD1

Tuesday 2 November 2004 Afternoon Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Questions 1, 4, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach the insert to your answer book.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

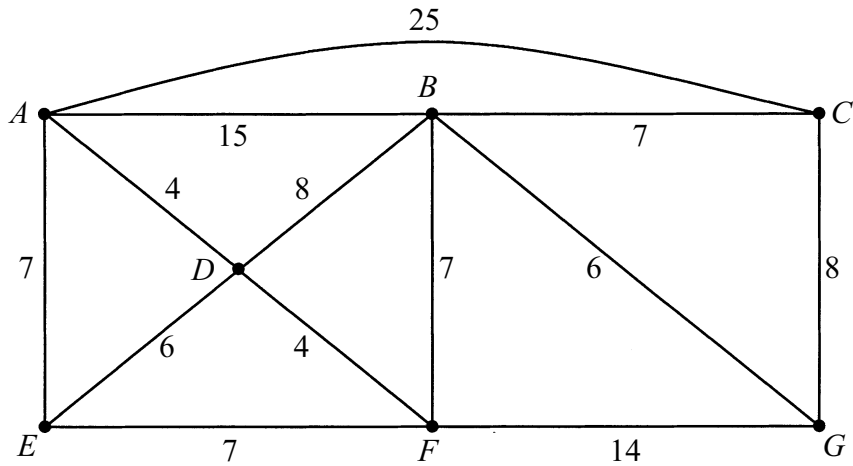
Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 [Figure 1, printed on the insert, is provided for use in answering part (a) of this question.]

The network shows seven towns $A-G$, the water pipes which connect them, and the lengths of those pipes in miles.



- (a) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest total length of pipes connecting A to G . Show all your working at each vertex. (5 marks)
- (ii) Deduce which pipes form the shortest route from A to G , making your working clear. (2 marks)
- (b) The water company is planning to replace some of the pipes. Use Prim's algorithm, starting at A , to find the minimum total length of pipes which must be replaced so that any two of the towns are interlinked by replaced pipes. (5 marks)
- (c) Draw a network representing the minimum connector which you found in part (b) and state the distance from A to G in your network. (2 marks)
- (d) The water company decides that each town should be interlinked to A by replaced pipes of total length less than 20 miles.
- (i) Show that for town G this means that the pipes AD , DB and BG must all be replaced. (1 mark)
- (ii) State which further pipes must be replaced in order for the water company to achieve its aim with the minimum possible total length of replaced pipes. (3 marks)

2 Use truth tables to show that

$$\sim p \wedge \sim(p \Rightarrow q)$$

is a contradiction.

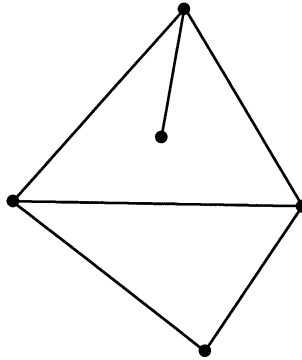
(5 marks)

3 A graph consists of six vertices labelled 5, 6, 8, 9, 10 and 12. Two vertices are joined by an edge if their labels have a common factor larger than 1. So, for example, vertex 9 is joined to only vertices 6 and 12.

(a) Draw the graph.

(4 marks)

(b) State which vertex must be removed in order to leave a graph which is isomorphic to the following:



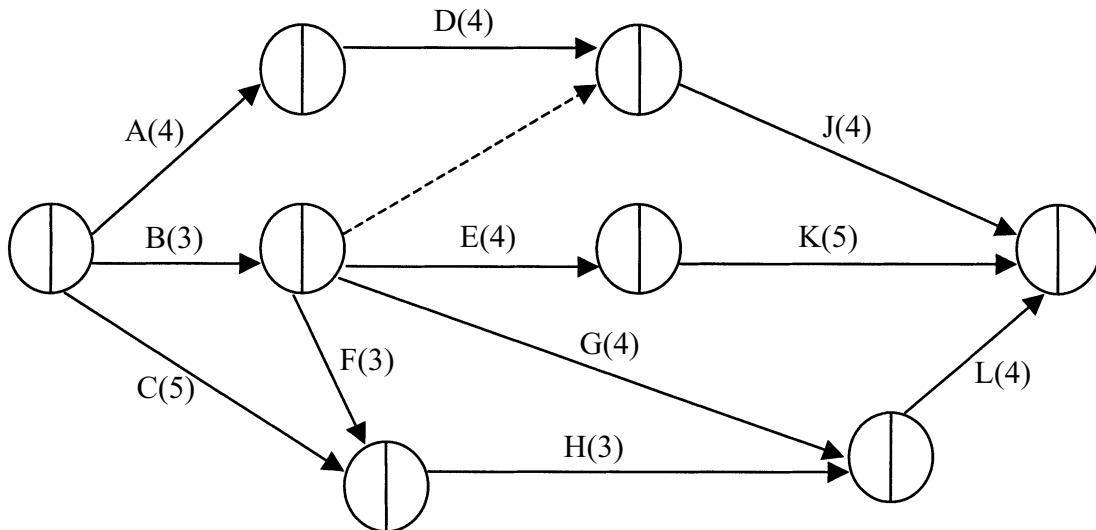
(2 marks)

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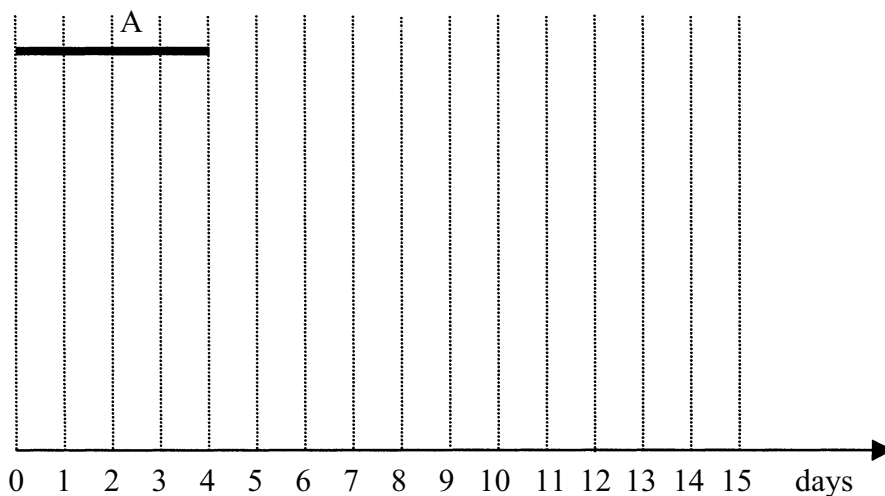
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- 4 [Figures 2 and 3, printed on the insert, are provided for use in answering parts (b) and (d) of this question.]

The following activity network of a complex project involves activities A, B, C, D, E, F, G, H, J, K and L, whose durations are given in days.



- (a) List all the activities which must be completed before activity J can start. (1 mark)
- (b) Perform a forward and backward pass on **Figure 2** in order to calculate all the early and late event times. (4 marks)
- (c) Find the critical path and the minimum completion time of the project. (2 marks)
- (d) The following diagram begins to illustrate a schedule for the project to be completed in the minimum possible time with each activity starting as early as possible.



On **Figure 3**, complete this schedule.

(4 marks)

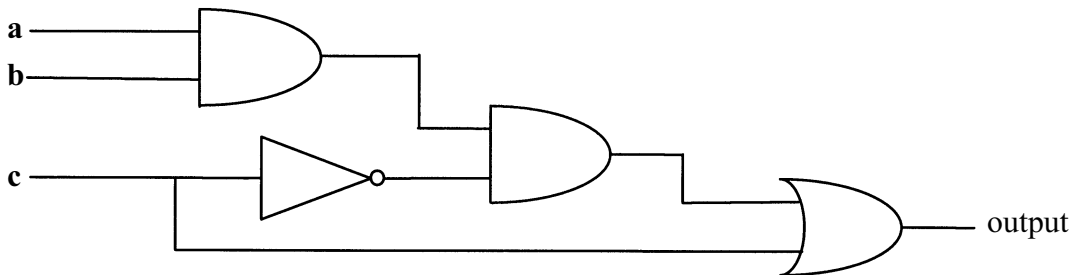
(e) The activities require the following numbers of workers:

Activity	A	B	C	D	E	F	G	H	J	K	L
Number of workers	2	5	1	2	3	1	2	1	2	3	3

- (i) Calculate the minimum number of workers needed for your schedule in part (d).
(3 marks)
- (ii) By calculating the total number of working days required, show that it is impossible to complete the project in the minimum time with just seven workers. (2 marks)
- (iii) State which single activity should be delayed in order for the project to be completed in the minimum time with just eight workers. State how long this delay should be.
(2 marks)

5 [Figure 4, printed on the insert, is provided for use in answering part (a) of this question.]

A combinatorial circuit requires three inputs **a**, **b** and **c** to produce an output, as shown:



- (a) By tracing an output of 0 back through the circuit on **Figure 4**, or otherwise, find the three sets of values of **a**, **b** and **c** which give a zero output.
(5 marks)
- (b) Draw a combinatorial circuit with only two gates which is equivalent to the circuit above.
(4 marks)

- 6 [Figure 5, printed on the insert, is provided for use in answering part (b) of this question. The figure consists of graph paper with the axes already drawn.]

An athlete wants to supplement his diet with vitamins. He can buy either Xtravim tablets or Yeasty tablets.

Each Xtravim tablet contains 3 mg of vitamin A, 1 mg of vitamin B and 1 mg of vitamin C.

Each Yeasty tablet contains 1 mg of vitamin A, 4 mg of vitamin B and 1 mg of vitamin C.

The athlete's minimum daily requirement is 15 mg of vitamin A, 20 mg of vitamin B and 11 mg of vitamin C.

Assume that the athlete takes x Xtravim tablets and y Yeasty tablets each day.

- (a) Show that, in order to satisfy the athlete's requirement of vitamin A, x and y must satisfy

$$3x + y \geq 15$$

and write down the corresponding inequalities for the vitamin B and the vitamin C.

(3 marks)

- (b) On **Figure 5**, illustrate the region of those (x, y) which satisfy $x \geq 0$, $y \geq 0$ and the three inequalities in part (a). (4 marks)

- (c) The Xtravim tablets cost $2p$ each and the Yeasty tablets cost $5p$ each.

- (i) Find how many tablets of each type the athlete should take each day in order to satisfy his vitamin requirements at minimum cost. (4 marks)

- (ii) The Xtravim tablets are also available more cheaply by mail order. The new minimum cost to the athlete can still be achieved by continuing to take the same mixture of tablets as before or by taking a sufficient number of Xtravim tablets alone. Find the cost of Xtravim tablets when bought by mail order. (4 marks)

- 7 A simple graph has six vertices and their degrees are $d, d + 3, d + 3, d + 3, d + 3$ and $d + 4$.

- (a) (i) Explain why d must be either 0 or 1. (1 mark)

- (ii) For which of these values of d will the graph be semi-Eulerian? Give a reason. (2 marks)

- (iii) Show that the graph will never be Hamiltonian. (2 marks)

- (b) Draw the graph in the case when $d = 1$ and state whether it is planar or not, giving a reason. (4 marks)

END OF QUESTIONS

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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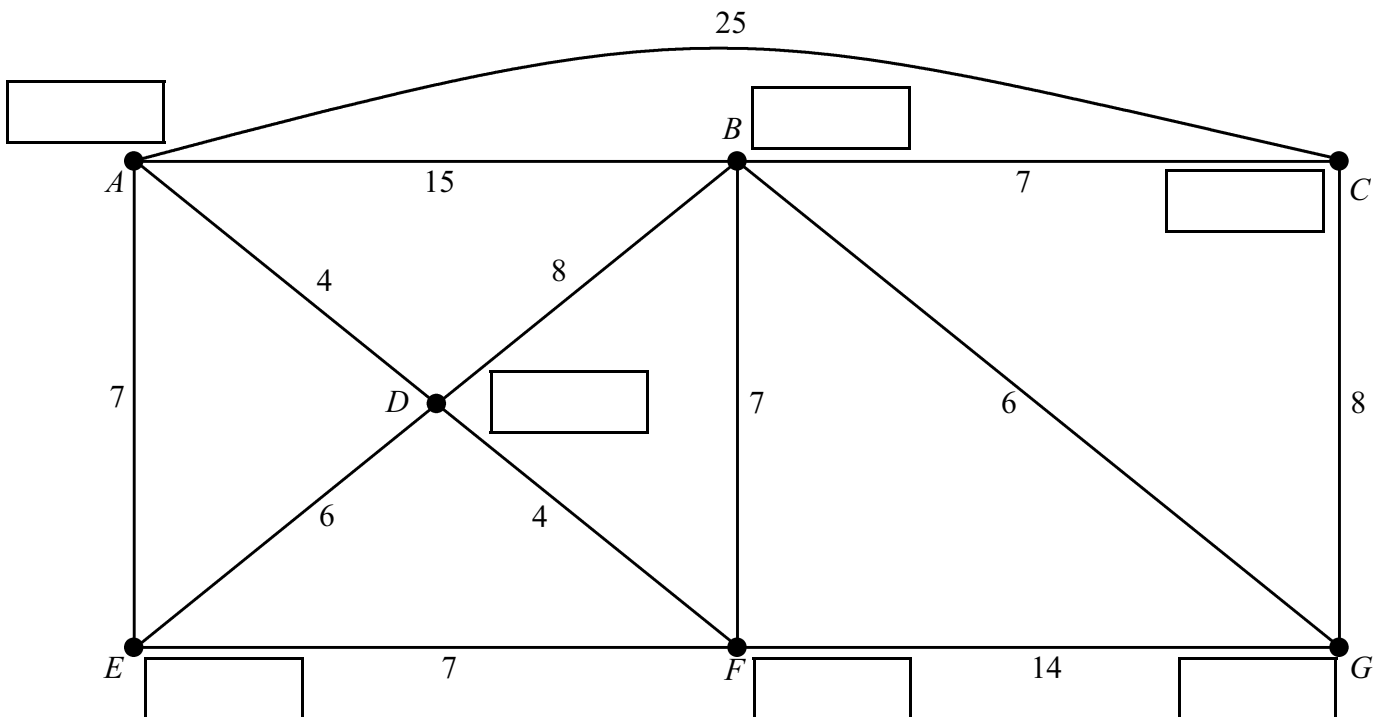


Figure 1

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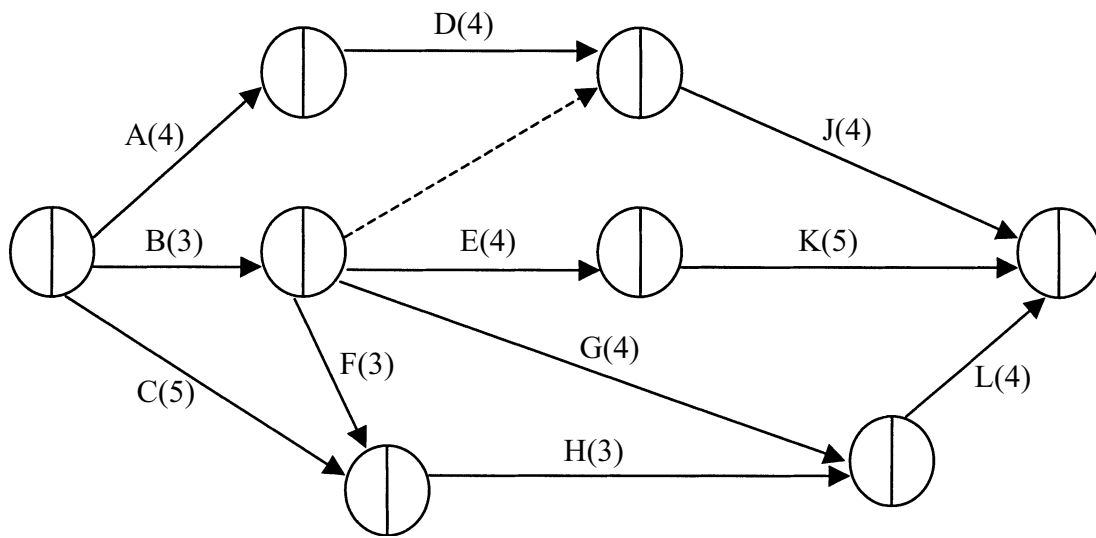


Figure 2

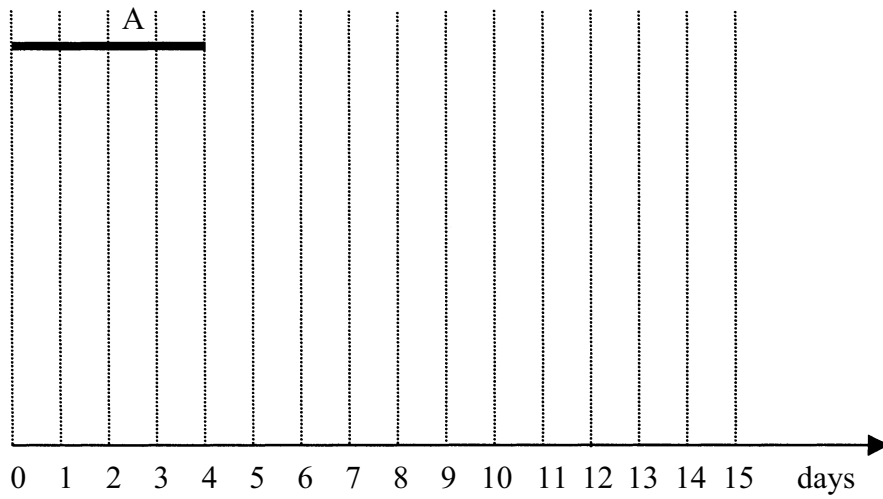


Figure 3

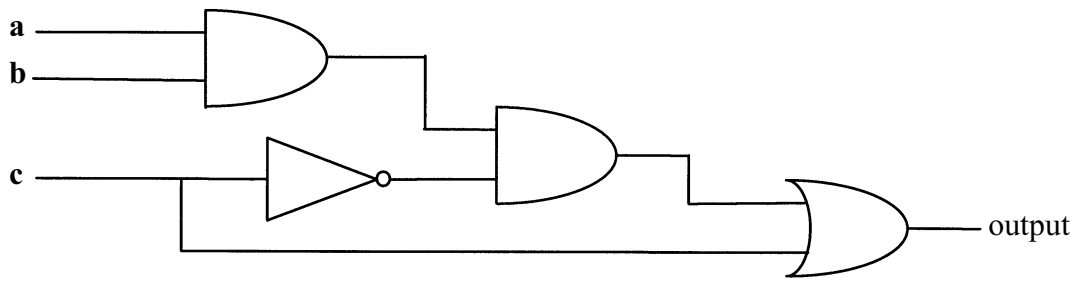


Figure 4

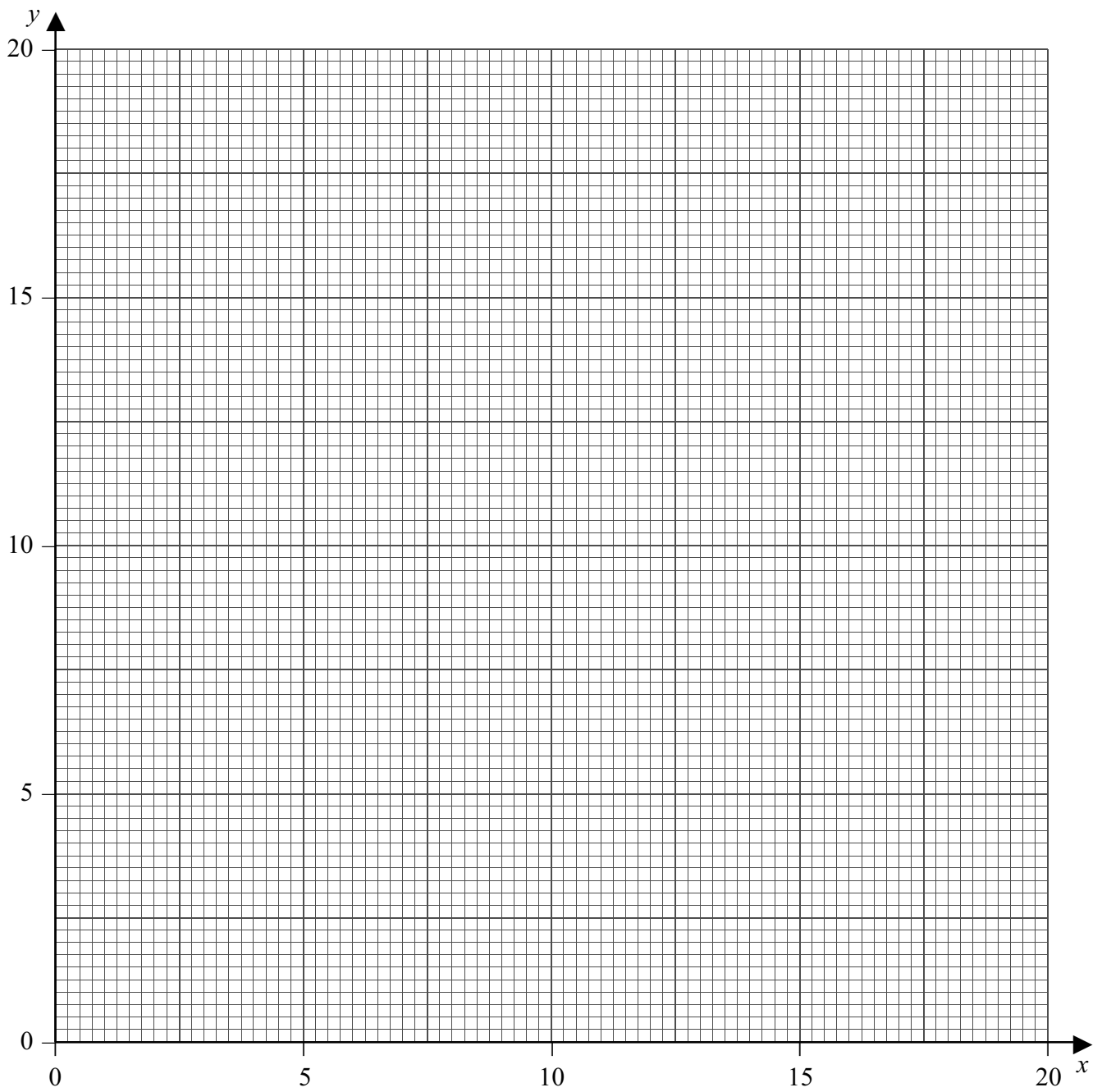


Figure 5