# GCE 2004
*June Series*

**AQA**

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

# Mark Scheme

# Mathematics and Statistics B
*MBS7*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key to Mark Scheme

| | | |
|---|---|---|
| **M** | mark is for | method |
| **m** | mark is dependent on one or more M marks and is for | method |
| **A** | mark is dependent on M or m marks and is for | accuracy |
| **B** | mark is independent of M or m marks and is for | accuracy |
| **E** | mark is for | explanation |
| **√or ft or F** | | follow through from previous incorrect result |
| **cao** | | correct answer only |
| **cso** | | correct solution only |
| **awfw** | | anything which falls within |
| **awrt** | | anything which rounds to |
| **acf** | | any correct form |
| **ag** | | answer given |
| **sc** | | special case |
| **oe** | | or equivalent |
| **sf** | | significant figure(s) |
| **dp** | | decimal place(s) |
| **A2,1** | | 2 or 1 (or 0) accuracy marks |
| **–$x$ ee** | | deduct $x$ marks for each error |
| **pi** | | possibly implied |
| **sca** | | substantially correct approach |

## Abbreviations used in Marking

| | | |
|---|---|---|
| **MC – $x$** | | deducted $x$ marks for mis-copy |
| **MR – $x$** | | deducted $x$ marks for mis-read |
| **isw** | | ignored subsequent working |
| **bod** | | given benefit of doubt |
| **wr** | | work replaced by candidate |
| **fb** | | formulae book |

## Application of Mark Scheme

No method shown:

| | |
|---|---|
| **Correct answer without working** | **mark as in scheme** |
| **Incorrect answer without working** | **zero marks unless specified otherwise** |

More than one method / choice of solution:

| | |
|---|---|
| **2 or more complete attempts, neither/none crossed out** | **mark both/all fully and award the mean mark rounded down** |
| **1 complete and 1 partial attempt, neither crossed out** | **award credit for the complete solution only** |

| | |
|---|---|
| Crossed out work | **do not mark unless it has not been replaced** |

| | |
|---|---|
| Alternative solution **using a correct or partially correct method** | **award method and accuracy marks as appropriate** |

**Mathematics and Statistics B Statistics 7 MBS7 June 2004**

| Question Number and Part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1 (a) | $\sum x = 21$  $\sum x^2 = 44.2314$ | | | |
| | $s = 0.12083$  $s^2 = 0.0146$ $(n-1)s^2 = 0.1314$  $\sigma^2 = 0.01314$ | B1 | | awrt 0.121  cao 0.0146 awrt 0.131  awrt 0.0131 |
| | $H_0$: $\sigma^2 = 0.01$ $H_1$: $\sigma^2 \neq 0.01$ | B1 | | Both Must be population parameters |
| | SL  $\alpha = 0.05$ DF  $v = 10 - 1 = 9$ | B1 | | cao |
| | CV  $\chi^2 = (2.7)$ and $19.023$ **or** CV  $F = 2.114$ | B1 | | Accept 19.0 but not 19  awrt 2.11 |
| | $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$  **or**  $F = \dfrac{s^2}{\sigma^2}$ = | M1 | | Use of; accept use of $s$ and/or $\sigma$ or $\sigma^4$ |
| | $\dfrac{9 \times 0.0146}{0.01} = 13.14$ **or** $\dfrac{0.0146}{0.01} = 1.46$ | A1 | | awfw 13.1 to 13.2  cao 1.46 |
| | Thus, at 5% level of significance, no evidence that value is not plausible | A1√ | 7 | **ag**; or equivalent ft on $\chi^2$ **or** $F$ and CV **sc** CI: B1 B0 B1 B1 M0 A0 A0 (max 3/7) |
| (b) | $\bar{x} = 2.1$ | B1 | | cao |
| | CI:  $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$ | M1 | | Accept  $\bar{x} \pm t\dfrac{s}{\sqrt{n}}$  or mixture |
| | $z = 2.5758$ | B1 | | awfw 2.57 to 2.58 |
| | CI:  $2.1 \pm 2.5758 \times \dfrac{0.1}{\sqrt{10}}$ | A1√ | | ft on $\bar{x}$ and $z$ but no mixture Accept $\pm 3.25 \times \dfrac{s}{\sqrt{10}}$; no ft on $t$ |
| | Thus  $(2.02, 2.18)$ | A1 | 5 | Dependent on use of correct $z$ **sc** Test: B1 M0 B1 A0 A0 (max = 2/5) |
| | **Total** | | **12** | |

**MBS7 (cont)**

| Question Number and Part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 2(a) | $SD(T) = 8$ | B1 | 1 | cao |
| (b) | $P(5 < T < 15) =$ | | | |
| | $P(T < 15) - P(T < 5)$ | M1 | | Use of; or use of $\int \lambda e^{-\lambda t}\, dt$ |
| | $= \left(1 - e^{-\frac{15}{8}}\right) - \left(1 - e^{-\frac{5}{8}}\right)$ | A1 | | Or $\left[- e^{-\lambda t}\right]_5^{15}$ <br> $\geq 1$ correct term |
| | $= e^{-\frac{5}{8}} - e^{-\frac{15}{8}}$ | | | |
| | $= 0.53526 - 0.15335 = 0.382$ | A1 | 3 | awrt |
| (c) | $P(\text{none from 5.45 to 6.00}) = P(T > 15)$ | M1 | | Attempt at either probability |
| | $= e^{-\frac{15}{8}} = 0.153$ | A1✓ | | ft on part (b)    (0.153355) |
| | $P(\text{none from 6.00 to 6.15}) = P(S > 15)$ | | | |
| | $= e^{-1} = 0.368$ | A1 | | cao/awrt          (0.367879) |
| | $P(\text{none from 5.45 to 6.15}) = \text{'product'}$ | m1 | | Use of |
| | $= e^{-\frac{23}{8}} = 0.056 \text{ to } 0.057$ | A1 | 5 | awfw |
| | **Or** (using Poisson) = | (M1) | | |
| | $P(0 | \lambda = \frac{15}{8} = 1.875)$ <br> $\times$ <br> $P(0 | \lambda = \frac{15}{15} = 1)$ | (A1) <br><br> (A1) | | |
| | $= e^{-1.875} \times e^{-1}$ | (m1) | | Product |
| | $= e^{-2.875} = 0.056 \text{ to } 0.057$ | (A1) | | awfw |
| | **Total** | | **9** | |

**MBS7 (cont)**

| Question Number and Part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 3(a)(i) | $H_0$: $p = 0.015$ (1.5%) | B1 | | Accept $\lambda = 1.5$ ⎤ cannot be |
| | $H_1$: $p > 0.015$ (1.5%) | B1 | 2 | Accept $\lambda > 1.5$ ⎦ scored later |
| (ii) | $\lambda = 1.5$ | B1 | | Stated or implied in (ii) |
| | $P(X \geq 4) = 1 - P(X \leq 3)$ | M1 | | Attempt at |
| | $= 1 - e^{-1.5}\left(1 + 1.5 + \dfrac{1.5^2}{2} + \dfrac{1.5^3}{6}\right)$ | M1 | | Attempt at $P(X \leq 3)$ for Po(1.5) |
| | $= 1 - 0.22313 \times 4.1875 = 0.065$ to $0.066$ | A1 | | awfw |
| | $< 0.10$ (10%) | M1 | | Comparison with 10% |
| | Thus, at 10% level of significance, reason to accept wholesaler's suspicion. | A1√ | 6 | ft probability with 10% |
| (b) | Normal approximation with: | | | |
| | $\mu = 30$ and $\sigma^2 = 29.55$ or $30$ or $\hat{p} = 0.018$ | B1 | | Both; cao, awfw 29.5 to 30 cao |
| | CV $\quad z = 1.2816$ | B1 | | awrt 1.28 |
| | $z = \dfrac{(36 \text{ or } 35.5) - 30}{\sqrt{29.55 \text{ or } 30}}$ or $\dfrac{0.018 - 0.015}{\sqrt{\dfrac{0.015 \times 0.985}{2000}}}$ | M1 | | Normal standardisation |
| | 1.00 to 1.15 | A1 | | awfw; (*p*-value = 0.125 to 0.160) |
| | $P(X \geq 36 \mid B(2000, 0.015)) = 0.155669$ | (M1) | | |
| | $P(X \geq 36 \mid Po(30)) = 0.157383$ | (A2) | | |
| | Comparison with 0.10 (10%) | (m1) | | |
| | Thus, at 10% level of significance, no reason to accept wholesaler's suspicion | A1√ | 5 | ft on *z* and CV or ft on probability with 10% |
| | **Total** | | **13** | |

**MBS7 (cont)**

| Question Number and Part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 4(a) | **Linear** relationship | E1 | | Or equivalent |
| | **Negative** relationship | E1 | 2 | Or equivalent |
| (b) | $\hat{\beta} = \dfrac{-46.5}{6938} = -0.0067$ to $-0.006705$ | B1 | | awfw |
| | $\hat{\alpha} = 6.55 + 0.006702 \times 44 = 6.84$ to $6.85$ | B1 | 2 | awfw |
| (c)(i) | $s^2 = \dfrac{1}{12-2}\left(0.3268 - \dfrac{(-46.5)^2}{6938}\right)$ | M1 | | Attempt at |
| | $= 0.00151$ to $0.00152$ | A1 | 2 | awfw |
| (ii) | $H_0: \beta = -0.005$ <br> $H_1: \beta < -0.005$ | B1 | | Both |
| | SL $\qquad \alpha = 0.01$ <br> DF $\qquad \nu = 12 - 2 = 10$ | B1 | | cao |
| | CV $\qquad t = -2.764$ | B1 | | awrt $\pm 2.76$ |
| | $t = \dfrac{\hat{\beta} - \beta_0}{\sqrt{\dfrac{s^2}{S_{xx}}}} = \dfrac{-0.0067 - (-0.005)}{\sqrt{\dfrac{0.0015146}{6938}}}$ | M1 | | Use of; allow $\beta_0 = 0$ but not $\sqrt{n}$ |
| | | A1√ | | ft on $\hat{\beta}$ and $s^2$ |
| | $= -3.66$ to $-3.63$ | A1 | | awfw; ignore sign |
| | Thus, at 1% level of significance, evidence that $\beta < -0.005$ | A1√ | 7 | ft on $t$ and CV providing consistent signs |
| (iii) | For every 1°C **rise** in temperature | B1 | | Or equivalent |
| | the pH **decreases**, on average, by **more than 0.005** | B1√ | 2 | Or equivalent <br> ft on part (c)(ii) |
| | **Total** | | 15 | |

**MBS7 (cont)**

| Question Number and Part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 5(a)(i) | $P(X < x) = P\left(Z < \dfrac{x - 200}{\sqrt{100}}\right)$ $= P(Z < (0.1x - 20))$ | B1 | 1 | Accept 10 rather than $\sqrt{100}$ **ag** |
| (ii) | **Interval**          **Probability** $x < 180$                    0.02275 $180 < x \le 190$          0.13591 $190 < x \le 200$  given  (0.34134) $200 < x \le 210$          0.34134 $210 < x \le 220$          0.13591 $x \ge 220$                    0.02275 | B2, 1 | 2 | awfw 0.0227 to 0.0228 awrt 0.136 awrt 0.341 awrt 0.136 awfw 0.0227 to 0.0228 5 or 4 $\Rightarrow$ B2      3 or 2 $\Rightarrow$ B1 Cannot be scored in (b) |
| (b) | $H_0$: $X \sim N(200, 100)$ $H_1$: not $H_0$ | B1 | | or equivalent Not required |
| | SL        $\alpha = 0.05$ DF        $v = 6 - 1 = 5$ | B1 | | cao |
| | CV        $\chi^2 = 11.070$ | B1 | | awrt 11.1 |
| | $\begin{array}{ll} O & E \\ 19 & 13.65 \\ 74 & 81.55 \\ 193 & 204.80 \\ 218 & 204.80 \\ 85 & 81.55 \\ 11 & 13.65 \\ 600 & 600.00 \end{array}$ | M1  M1 | | Use of probabilities in part (a)(ii)  Probabilities $\times$ 600 |
| | $\chi^2 = \sum \dfrac{(O - E)^2}{E}$ | M1 | | Use of |
| | $= 4.90$ to $5.10$ | A1 | | awfw |
| | Thus, at 5% level of significance, no reason to reject hypothesis that $X \sim N(200, 100)$ | A1✓ | 8 | Or equivalent  ft on $\chi^2$ and upper CV **sc** If (a)(ii) not attempted or not used, then (b): B1 B1 B1 M0 M1 M1 A0 A1✓ (max 6/8) |
| | **Total** | | **11** | |
| | **TOTAL** | | **60** | |