General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBP7**
(SPECIFICATION B)
Unit Pure 7

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP7.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) For $0 \leqslant x \leqslant 2\pi$, draw the graph of $y = 1 + \cos 3x$. *(2 marks)*

(b) Sketch the curve with polar equation $r = 1 + \cos 3\theta \quad (0 \leqslant \theta \leqslant 2\pi)$. *(4 marks)*

2 (a) Show that $(a - c)$ is a factor of

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

and factorise Δ completely. *(5 marks)*

(b) Hence, or otherwise, show that the three planes

$$\begin{aligned} x - 2y + z &= 10 \\ 5x + 7y + 5z &= 21 \\ 12x + 10y + 12z &= 35 \end{aligned}$$

do not meet at a single point. *(2 marks)*

3 (a) Show that the series expansion of $e^x + \sin x$ is

$$1 + 2x + \tfrac{1}{2}x^2 + px^3 + qx^4 + \ldots$$

stating the value of p and the value of q. *(2 marks)*

(b) Given that

$$(1 + ax)^n = 1 + 2x + \tfrac{1}{2}x^2 + kx^3 + \ldots$$

where a, n and k are constants:

(i) find the value of a and the value of n; *(4 marks)*

(ii) determine the value of k; *(1 mark)*

(iii) state the range of values of x for which the expansion of $(1 + ax)^n$ is valid. *(1 mark)*

4 A rectangular hyperbola is represented parametrically by

$$x = 2t, \quad y = -\frac{2}{t}, \quad t \neq 0$$

(a) Show that the normal to this hyperbola has equation

$$y + t^2 x = \frac{2}{t}\left(t^4 - 1\right)$$

(4 marks)

(b) The normal meets the x-axis at P and the y-axis at Q. The midpoint of PQ is M. Determine a cartesian equation for the locus of M as t varies. You need not simplify your answer. *(5 marks)*

5 The group G consists of the set of functions f, f^2, $\mathrm{f}^3, \ldots,$ under the operation of composition of functions, defined for complex numbers z by

$$\mathrm{f}(z) = \mathrm{f}^1(z) = \mathrm{i}z + \mathrm{i}$$
$$\text{and } \mathrm{f}^n(z) = \mathrm{f}\{\mathrm{f}^{n-1}(z)\} \text{ for } n \geqslant 2$$

(a) (i) Show that $\mathrm{f}^4(z) = z$. *(4 marks)*

(ii) Hence describe the group G as fully as possible. *(2 marks)*

(b) State Lagrange's Theorem for groups of finite order, and use it to find possible orders for subgroups of G. For each possible order, describe the corresponding subgroup. *(3 marks)*

6 (a) A line has vector equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$.

(i) Explain the significance of the vectors **a** and **d** in relation to the line. *(1 mark)*

(ii) By considering a suitable vector product, show that the equation of the line can also be expressed in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$. *(2 marks)*

(b) Two lines have equations

$$(\mathbf{r} - (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})) \times (2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) = \mathbf{0}$$
$$\text{and } (\mathbf{r} - (\mathbf{i} + 2\mathbf{j} + \mathbf{k})) \times (\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = \mathbf{0}$$

(i) Determine $(2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) \times (\mathbf{i} - 3\mathbf{j} + \mathbf{k})$. *(2 marks)*

(ii) Find, in surd form, the shortest distance between these lines. *(3 marks)*

Turn over ▶

7 (a) In the complex plane, the circle C is described by the equation

$$|z - 2 + \mathrm{i}| = \sqrt{3}$$

(i) State the complex number which is the centre of C. *(1 mark)*

(ii) Sketch C on an Argand diagram. *(1 mark)*

(iii) Write down a cartesian equation for C. *(1 mark)*

(b) The half-line H has cartesian equation

$$y = mx - 1, \quad x > 0$$

You are given that $m > 0$ and that H is a tangent to C.

(i) Sketch H on your Argand diagram. *(1 mark)*

(ii) Show that $m = \sqrt{3}$. *(4 marks)*

(iii) Express the locus of H in the form $\arg(z - \alpha) = \theta$. *(2 marks)*

(iv) Determine the complex number which is represented by the point of contact of H and C. *(3 marks)*

END OF QUESTIONS