GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B

MBP5

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

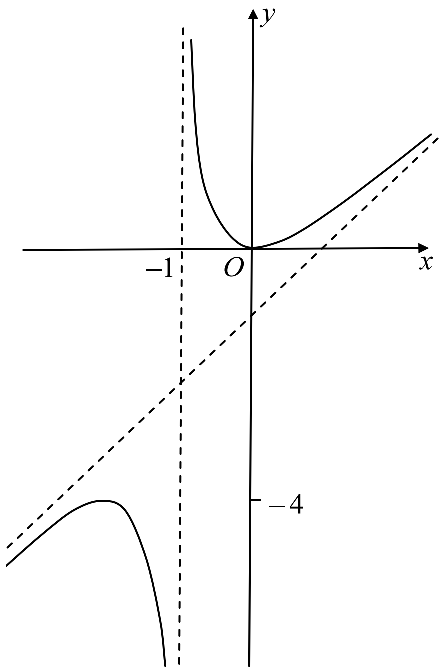
Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

Mathematics and Statistics B Pure 5 MBP5 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1(a)(i)	$(1+x)^{-1} \approx 1 - x.....$	B1		
	$+x^2 - x^3...$	B1	2	
(ii)	$(1+4x)^{\frac{1}{2}} = 1 +/- kx...$	M1		Valid start to binomial expn.
	$\left(\left(\frac{1}{2}\right)(4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(4x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(4x)^3 + ...\right)$	A1		Unsimplified form condone 1 error
	$= 1 + 2x - 2x^2 + 4x^3.....$	A2,1	4	(A1 if 3 simplified terms correct)
(b)	$...= 2(1+4x)^{\frac{1}{2}} + 4(1+x)^{-1} = ...$	M1		Clear correct use of both expansions
	$= 6 + 4x^3$	A1	2	cso Must be convinced
	Total		**8**	
2	Mid-ordinates are 1.25, and 1.75	B1		oe
	$\int \approx 0.5[\text{ f}(1.25) + \text{f}(1.75)]$	M1		
	$\approx 0.5\{0.17885098..+ 0.0622753..\}$ $\approx 0.1205.... = 0.121$ (to 3 dp)	A1	3	must be 0.121
	Total		**3**	
3(a)	$R\cos\alpha = 6$ or $R\sin\alpha = 8$ or $\tan\alpha = \frac{8}{6}$	M1		Accept seen; condone negative signs
	$R^2 = 6^2 + 8^2$	M1		Alternatively use 2 results in line1.
	$\Rightarrow$ $10\cos(x + 53.1°)$	A1	3	For α accept awrt 53.1° but R must be 10.
(b)	$\cos(x + \alpha) = 3/R$	M1		Possibly implied
	$x + \alpha = 360n \pm .....$	m1		Accept degrees, rads., mix
	$x + \alpha = 360n \pm 72.5(4...)$	A1		oe but right hand side must be in degrees
	$x = 360n \pm 72.5(4...) - 53.1(3..)$ [$x = 360n + \{19.3$ to 19.5 inclusive$\}$ $x = 360n - \{125$ to 126 inclusive$\}$]	A1	4	Any equivalent forms in degrees **sc** if m0 then award B1 for either one general solution or 2 particular solutions. covering both branches
	Total		**7**	

MBP5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dy}{dx} = \frac{1}{6}$	B1	1	
(ii)	Gradient of normal $= -6$	M1		ft on (i); used $m \times m' = -1$
	Eqn of normal $y-1=-6(x-1)$	m1		
	$y-1=-6x+6 \Rightarrow y+6x=7$	A1	3	**ag**
(b)	$2y\ dy = \frac{1}{x+2}\ dx$	M1		Clear attempt to separate variables appropriately
	$y^2 = \int \frac{1}{x+2} dx$	A1		
	$y^2 = \ln \lvert x+2 \rvert + c$	A1		Condone absence of mod signs and '$+c$'
	When $x = 1, y = 1 \Rightarrow 1 = \ln 3 + c$	m1		Valid method for c
	$y^2 = \ln (x + 2) + 1 - \ln 3$	A1	5	oe
	Total		**9**	
5(a)	Asymptote $x = -1$	B1		
	$y = x-1+\frac{1}{x+1}$	M1		Full attempt to divide out
	Asymptote $y = x - 1$	A1	3	
(b)	Turning point (0,0)	B1		**<u>Alternative</u>** Valid method to find $y'(x)$ and then puts $y'(x) = 0$ [M1]
	When $y = -4$, $x^2+4x+4=0$	M1		$x^2 + 2x = 0 \Rightarrow$ TPs (0,0) [A1]
	Turning point $(-2, -4)$	A1	3	and $(-2, -4)$ [A1]
(c)		B1		Single upper branch; shape and y not < 0
		B1		Single lower branch; shape and y not > -4
		B1	3	Dependent on previous two Bs. Asymptotic behaviour on both branches; through the origin
	Total		**9**	

MBP5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$\cos\theta = \dfrac{\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}}{\sqrt{1^2+(-1)^2+2^2}.\sqrt{2^2+2^2+1^2}}$	M1 B1		Use of a valid formula For either term in the denominator
	$\cos\theta = \dfrac{2}{\sqrt{6}.\sqrt{9}}$	A1	3	
(b)(i)	Angle between the normals to the planes $= \cos^{-1}\dfrac{2}{\sqrt{6}.\sqrt{9}}$	M1		
	Acute angle between the planes = 74.2°	A1√	2	Only ft on one arithmetical slip and A0 in (i). Accept nearest degree
(ii)	Subst. (−5, 3, 4) into Π_1 gives − 5 − 3 + 8 = 0 so (−5, 3, 4) lies on Π_1 Subst. (−5, 3, 4) into Π_2 gives −10 + 6 + 4 = 0 so (−5, 3, 4) lies on Π_2	B1	1	**ag**
(iii)	$\mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \lambda\overrightarrow{OA}$	M1		Clear understanding that the required line is *OA*, where *A* is (−5, 3, 4).
	$\mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}-5\\3\\4\end{pmatrix}$	A1	2	**oe**
	Total		**8**	

MBP5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	At P, $1 + \cos t = 0$	M1		
	$\Rightarrow t = \pi \Rightarrow x$-coordinate of P is π^2	A1	2	**ag**
(b)	$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = -\sin t$	M1		Attempts both and at least one correct (possibly implied)
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin t}{2t}$	A1	2	
(c)(i)	$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{-\sin t}{2t}\right)$			
	$= \left[\frac{2t(-\cos t) - (-\sin t)2}{4t^2}\right]$	M1		quotient rule oe used
	$= \frac{\sin t - t\cos t}{2t^2}$	A1	2	**ag** cso
(ii)	$\frac{d^2y}{dx^2} = \frac{dt}{dx}\frac{d}{dt}\left(\frac{dy}{dx}\right)$	M1		using valid formula
	$= \frac{1}{2t}\left[\frac{\sin t - t\cos t}{2t^2}\right]$	A1	2	
(iii)	Necessary condition for pt. of inflection			
	$\frac{d^2y}{dx^2} = 0 \Rightarrow \sin t - t\cos t = 0$			
	$\Rightarrow \frac{\sin t}{\cos t} = t \Rightarrow \tan t = t$	E1	1	**ag**
(d)(i)	$\int t\cos t\, dt = t(\sin t) - \int(\sin t)dt$	M1		Condone sign errors only
		A1		cao
	$= t\sin t + \cos t + c$	A1√	3	ft on previous result Condone absence of $+c$
(ii)	Shaded area = $\int_0^\pi y\frac{dx}{dt}dt$	M1		Need attempt to write integrand in terms of t. (ignore limits)
	….. = $\int_0^\pi (1 + \cos t)2t\, dt$	A1 B1		Ignore limits For correct limits seen
	Shaded area = $\pi^2 - 4$	A1	4	Award for correct solution only
	Total		**16**	
	TOTAL		**60**	