General Certificate of Education
June 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Pure 3

MBP3

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the inverse of the matrix $\boldsymbol{A} = \begin{bmatrix} 3 & -5 \\ 4 & 1 \end{bmatrix}$. *(2 marks)*

(b) Use the inverse matrix to solve the simultaneous equations

$$3x - 5y = 11$$
$$4x + y = 7$$ *(3 marks)*

2 (a) A curve has equation $y = \dfrac{3x + 4}{1 - 2x}$.

(i) Find the coordinates of the points where the curve crosses the coordinate axes. *(2 marks)*

(ii) State the equations of its asymptotes. *(2 marks)*

(iii) Sketch the curve. *(2 marks)*

(b) Calculate the x-coordinate of the point where the curve $y = \dfrac{3x + 4}{1 - 2x}$ intersects the line $y = 1$. *(2 marks)*

(c) Hence, or otherwise, solve the inequality $\dfrac{3x + 4}{1 - 2x} \leqslant 1$. *(3 marks)*

3 The roots of the quadratic equation $x^2 + (7 + p)x + p = 0$ are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$, in terms of p. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$ in terms of p. *(2 marks)*

(c) (i) Show that $(\alpha - \beta)^2 = p^2 + 10p + 49$. *(2 marks)*

(ii) Given that α and β differ by 5, find the possible values of p. *(3 marks)*

4 (a) Find the modulus and argument of the complex number $-1 + \sqrt{3}\,\mathrm{i}$, giving the argument in terms of π. *(4 marks)*

(b) Prove that $(-1 + \sqrt{3}\,\mathrm{i})^3$ is real. *(3 marks)*

(c) Given that $-1 + \sqrt{3}\,\mathrm{i}$ is a root of the equation $z^3 + k = 0$, where k is real:

(i) state the value of k; *(1 mark)*

(ii) write down another complex root of the equation $z^3 + k = 0$. *(1 mark)*

5 The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are such that

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(a) Find the matrix product $\boldsymbol{AB}$. *(2 marks)*

(b) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Describe fully the geometrical transformation represented by **T** for each of the following cases.

(i) $\boldsymbol{M} = \boldsymbol{A}$ *(2 marks)*

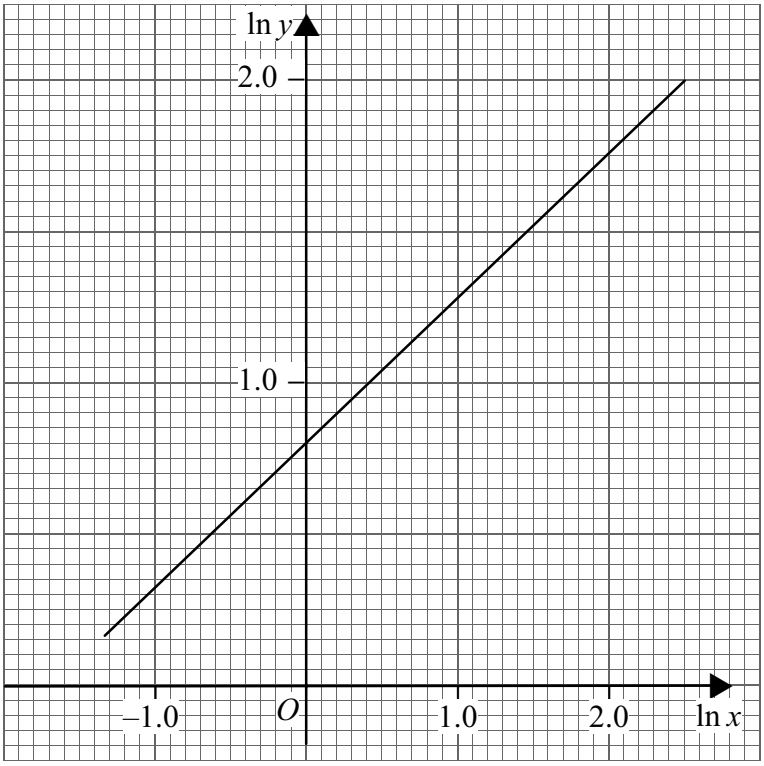
(ii) $\boldsymbol{M} = \boldsymbol{B}$ *(2 marks)*

(iii) $\boldsymbol{M} = \boldsymbol{AB}$ *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

6 A student performs an experiment and records data for two variables x and y.

Values of $\ln x$ and $\ln y$ are calculated and a line of best fit is drawn as shown below.



(a) Use this graph to find the value of y when $x = 3.0$, giving your answer to two significant figures. *(3 marks)*

(b) The student believes there is a relationship between x and y of the form $y = Ax^n$, where A and n are constants.

(i) Express $\ln y$ in terms of $\ln A$, $\ln x$ and n. *(1 mark)*

(ii) Use the graph to estimate the values of A and n, giving your answers to two significant figures. *(4 marks)*

7 (a) Show that $\dfrac{4}{(k+2)(k+3)} - \dfrac{4}{(k+2)} \equiv \dfrac{-4}{(k+3)}$. *(2 marks)*

(b) Prove by mathematical induction that, for all positive integers $n \geqslant 1$,

$$\sum_{r=1}^{n} \frac{4}{(r+1)(r+2)} = 2 - \frac{4}{(n+2)}$$ *(4 marks)*

(c) Hence find:

(i) $\displaystyle\sum_{r=3}^{n} \frac{4}{(r+1)(r+2)}$; *(2 marks)*

(ii) $\displaystyle\sum_{r=3}^{\infty} \frac{4}{(r+1)(r+2)}$. *(1 mark)*

8 (a) (i) Describe a geometrical transformation that maps the curve with equation

$$x^2 + y^2 = 1$$

onto the curve with equation $(x-1)^2 + y^2 = 1$. *(2 marks)*

(ii) Sketch the curve with equation $(x-1)^2 + y^2 = 1$. *(2 marks)*

(b) Show that the curve with cartesian equation

$$(x-1)^2 + y^2 = 1 + 2\sqrt{x^2 + y^2}$$

has the equation $r = 2 + 2\cos\theta$, where $[r, \theta]$ are polar coordinates. *(3 marks)*

(c) For the curve with equation $r = 2 + 2\cos\theta$, where $[r, \theta]$ are polar coordinates:

(i) state the greatest and least values of r; *(2 marks)*

(ii) sketch its graph for $-\pi < \theta \leqslant \pi$. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

9 The binary operation $\otimes$ is defined for all integers a and b by

$$a \otimes b = a + b - 4$$

(a) Determine, with a reason, whether the set of integers is closed under $\otimes$. *(2 marks)*

(b) Find the identity element, e. *(2 marks)*

(c) Find the inverse of the element x. *(2 marks)*

(d) Prove that $\otimes$ is associative. *(3 marks)*

END OF QUESTIONS

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