General Certificate of Education
June 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP2**
(SPECIFICATION B)
Unit Pure 2

Friday 11 June 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Find, in its simplest form,

$$\int_2^6 \frac{1}{x}\,\mathrm{d}x$$

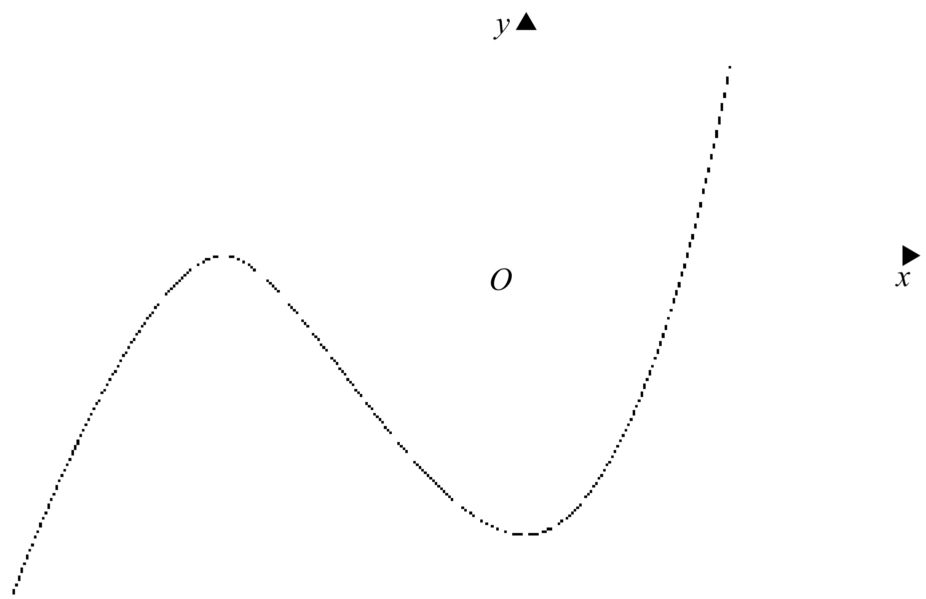
(3 marks)

2 The second term of a geometric series is 16 and the sixth term is 1.

(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{2}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{2}$, find the sum to infinity of the series. *(3 marks)*

3 The diagram shows part of the curve with equation $y = (x-1)(x+2)^2$.



(a) Write down the **two** values of x where $y = 0$. *(2 marks)*

(b) Solve the inequality

$$(x-1)(x+2)^2 > 0$$

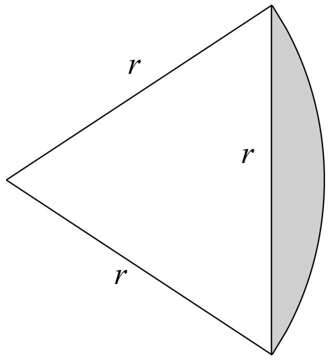
(1 mark)

(c) Hence solve the inequality

$$(x-1)(x+2)^2 \geqslant 0$$

(2 marks)

4 The diagram shows a shaded segment of a circle of radius r cm. The segment is formed by drawing an arc on one side of an equilateral triangle of side r cm with the centre at the opposite vertex.



(a) Show that the ratio of the length of the arc to the side of the triangle is $\pi:3$. *(2 marks)*

(b) Show that the area of the triangle is $\frac{\sqrt{3}}{4}r^2$ cm^2. *(2 marks)*

(c) Given that the area of the shaded segment is 10 cm^2, find, to 3 significant figures, the value of r. *(3 marks)*

5 (a) (i) Verify that $\frac{1}{4}x < \ln x$ when $x = 2$. *(1 mark)*

(ii) Verify that $\frac{1}{4}x > \ln x$ when $x = 10$. *(1 mark)*

(iii) Draw on the same diagram sketches of the graphs with equations $y = \frac{1}{4}x$ and $y = \ln x$ for $x > 0$. *(2 marks)*

(iv) Hence state the number of roots of the equation

$$\frac{1}{4}x = \ln x, \qquad x > 0$$ *(1 mark)*

(b) The curve, C, with equation

$$y = \ln x - \frac{1}{4}x, \qquad x > 0$$

has only one stationary point.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(iii) Find the x-coordinate of the stationary point. *(2 marks)*

(iv) Determine whether the stationary point is a maximum or a minimum. *(2 marks)*

Turn over ▶

6 (a) Given that $(x+2)$ is a factor of

$$p(x) = 6x^3 + kx^2 - 9x + 2$$

show that $k = 7$. *(2 marks)*

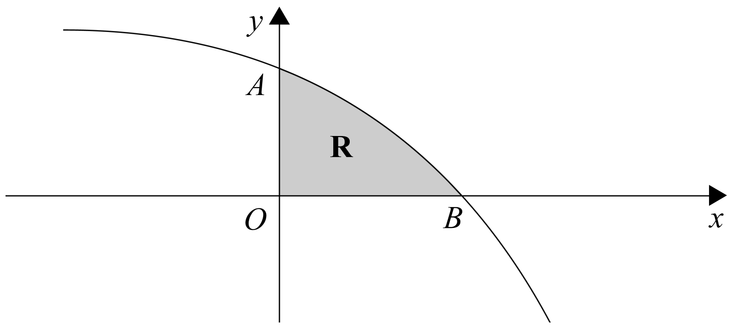
(b) Find the value of $p\left(\frac{1}{2}\right)$ and hence show that $(2x-1)$ is a factor of $p(x)$. *(2 marks)*

(c) Express $p(x)$ as a product of three linear factors. *(2 marks)*

(d) Hence find the values of θ, in radians, in the interval $0 < \theta < 2\pi$ for which

$$6\sin^3\theta + 7\sin^2\theta - 9\sin\theta + 2 = 0$$ *(6 marks)*

7 The diagram shows a sketch of the curve with equation $y = 8 - e^{3x}$ which crosses the y-axis at the point A and the x-axis at the point B.



(a) Find the y-coordinate of A. *(1 mark)*

(b) Find the **exact** value of the x-coordinate of B. *(2 marks)*

(c) Show that the gradient of the curve at B is -24. *(2 marks)*

(d) (i) Find $\int (8 - e^{3x})\, dx$. *(2 marks)*

(ii) Hence show that the area of the shaded region, **R**, bounded by the curve $y = 8 - e^{3x}$ and the coordinate axes is

$$8\ln 2 - \frac{7}{3}$$ *(3 marks)*

(e) (i) Sketch the graph of the curve $y = |8 - e^{3x}|$. *(2 marks)*

(ii) Solve the equation $|8 - e^{3x}| = 19$, giving your answer in an **exact** form. *(2 marks)*

END OF QUESTIONS