GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B

MBP2

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Publications Department, Aldon House, 39, Heald Grove, Rusholme, Manchester, M14 4NA
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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted x marks for mis-copy
MR – *x*	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

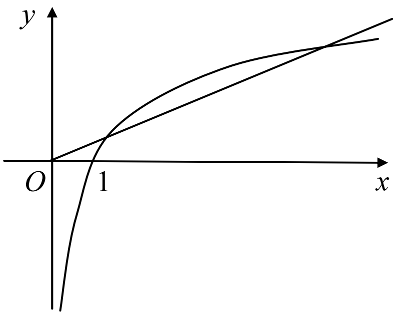
Mathematics and Statistics B Pure 2 MBP2 June 2004

Question number and Part	Solution	Marks	Total marks	Comments
1	$\int \frac{1}{x}\,dx = \ln x \ldots.$	B1		cao
	$\int_2^6 \frac{1}{x}\,dx = \ln 6 - \ln 2$	M1		Dealing with limits correctly; F(6)–F(2) [B0M1 possible following an attempt to integrate $\frac{1}{x}$.]
	$\ldots\ldots = \ln 3$	A1	3	cso (use ISW for dec following ln3)
	Total		**3**	
2(a)	$ar = 16; \quad ar^5 = 1$	B1		For either oe
	$\Rightarrow 16r^4 = 1$	M1		Elimination of a oe
	$r^4 = \frac{1}{16} \quad \Rightarrow r = -\frac{1}{2}$	A1		**ag** cso Full valid completion **sc** Clear explicit verification give maximum B2 out of 3. (accept if −8, 4, −2 seen)
	or $r = \frac{1}{2}$	B1	4	
(b)	$a = -32$	B1		
	$\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{2}\right)}$	M1		Accept $\frac{a}{1-r}$ quoted
	$S_\infty = \frac{-64}{3}$ (= – 21.3 to 3sf)	A1√	3	ft on candidate's value for a, ie $\frac{2}{3}a$ **sc** cand uses $r = 0.5$, gives $a = 32$ and sum to infinity = 64 (max. B0M1A1)
	Total		**7**	
3(a)	−2 and 1	B1B1	2	
(b)	$x > 1$	B1√	1	ft on $x >$ larger value in (a) if not $x > 1$
(c)	$x \geq 1$	B1√		ft on $x \geq$ larger value in (a) if not $x \geq 1$
	$x = -2$	B1√	2	ft on x = smaller value in (a) if not $x = -2$
	Total		**5**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
4(a)	Angle 60° or $\dfrac{\pi}{3}$ radians or arc = $r\theta$	B1		Seen in part (a)
	arc : radius = $\dfrac{60}{360}2\pi r : r = \pi : 3$	B1	2	**ag** cso
(b)	area of equilateral $\Delta = \dfrac{1}{2}r^2 \sin 60^\circ$	M1		$\dfrac{1}{2}r^2 \sin\theta$ oe stated
	$= \dfrac{1}{2}r^2 \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}r^2$	A1	2	**ag** cso
(c)	Area of sector = $\dfrac{1}{6}\pi r^2$	B1		
	$\dfrac{1}{6}\pi r^2 - \dfrac{\sqrt{3}}{4}r^2 = 10$	M1		Accept "area of sector $-\dfrac{\sqrt{3}}{4}r^2 = 10$"
	$r^2 = \dfrac{120}{2\pi - 3\sqrt{3}}$ = 110.3922….			
	r = 10.506…. = 10.5 to 3sf	A1	3	awrt 10.5
	Total		**7**	

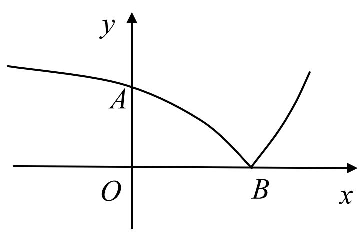
MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
5(a)(i)	$\frac{1}{4}(2) < \ln 2$ since $0.5 < 0.693..$	B1	1	Be convinced
(ii)	$\frac{1}{4}(10) > \ln 10$ since $2.5 > 2.30..$	B1	1	Be convinced
(iii)		B2,1	2	B2 graphs correct with detail. (B1 for full correct shape of $y = \ln x$; or for correct line **and** correct part graph of $y = \ln x$ in first quadrant from marked point (1,0))
(iv)	2 roots	B1	1	
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x} - \frac{1}{4}$	M1		One term correct
		A1	2	Accept other correct forms
(ii)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{1}{x^2}$	A1√	1	Only ft if $y'(x)$ has x^{-1} term
(iii)	At st. pt $y'(x) = 0 \Rightarrow \frac{1}{x} - \frac{1}{4} = 0$	M1		Putting their $y'(x) = 0$
	$\Rightarrow \quad x = 4$	A1	2	
(iv)	$x^2 > 0 \Rightarrow y''(x) < 0$ {alt. $y''(4) < 0$}	M1		Finding sign of 2nd derivative, or consideration of sign of $y'(x)$ either side of st pt or relevant use of (a) oe
	$\Rightarrow$ st pt is a maximum	B1	2	Stated
	Total		**12**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
6(a)	$(x+2)$ is a factor of $p(x) \Rightarrow p(-2)=0$	M1		Use of $p(-2)$ **<u>or</u>** if division by $x+2$, correctly reaches remainder $4k-28$ $\{Q(x)=6x^2+(k-12)x-2k+15\}$ oe comparing coefficients
	$-48+4k+18+2=0 \Rightarrow k=7$	A1	2	**ag** cso
(b)	$p\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 + k\left(\frac{1}{2}\right)^2 - 9\left(\frac{1}{2}\right) + 2$	M1		Use of $p\left(\frac{1}{2}\right)$. Accept $p\left(\frac{1}{2}\right)=0$ stated
	$=0 \Rightarrow (2x-1)$ is a factor of $p(x)$	A1	2	**ag** Must have the conclusion
(c)	$(x+2)(2x-1)[3x \ldots -1]$	M1		Valid attempt at 3rd factor (coeff of x^3 or const correct)
	$p(x) \equiv (x+2)(2x-1)(3x-1)$	A1	2	
(d)	$x \to \sin\theta \Rightarrow$			
	$(\sin\theta+2)(2\sin\theta-1)(3\sin\theta-1)=0$	M1		Using $x=\sin\theta$
	$\Rightarrow \sin\theta=-2; \Rightarrow$ no solution	B1√		PI
	$\sin\theta=\frac{1}{2}; \Rightarrow \theta=\frac{\pi}{6}$	A1		Accept 0.523 or 0.524 or better
	$\theta = \frac{5\pi}{6} = 2.62$ (3 sf)	A1√		ft on $\pi -$ "$\frac{\pi}{6}$" accept 3sf or better
	$\sin\theta=\frac{1}{3}; \Rightarrow \theta=0.339(8..)$	A1√		ft on cand's 3rd factor only if $\lvert\sin\theta\rvert \le 1$
	$\theta=2.80(17..)$	A1√	6	ft on $\pi -$ "0.339(8..)" Accept 2sf if 3rd sf is 0. Accept multiples of π. Ignore values outside the given interval. If answers are left in degrees deduct a maximum of 1 mark from A marks given.
	Total		**12**	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
7(a)	When $x = 0,\ y = 7$	B1	1	Accept 7 {seen at A on a sketch}
(b)	At B, $y = 0 \Rightarrow e^{3x} = 8$	M1		Reaching $e^{3x} = \pm 8$, PI by $x = 0.69(31..)$
	$\Rightarrow x = \frac{1}{3}\ln 8 \quad (= \ln 2)$	A1	2	Accept any correct exact form
(c)	$\frac{dy}{dx} = -3\,e^{3x}$	M1		For ke^{3x} ($k = \pm 3$ or $\pm\frac{1}{3}$). Condone any inclusion of '8'
	Gradient at $B = y'(\ln 2)$ $= -3\,(8) = -24$	A1	2	**ag** cso (must be exact throughout)
(d)(i)	$\int (8 - e^{3x})\,dx = 8x - \frac{1}{3}e^{3x} + c$	M1		For $8x - ke^{3x}$; $k = \frac{1}{3}$, 1 or 3 only
		A1	2	cao {condone absence of $+\,c$}
(ii)	$\int_0^{\ln 2} \left(8 - e^{3x}\right)dx = \left[8x - \frac{1}{3}e^{3x}\right]_0^{\ln 2}$			
	$= \left(8\ln 2 - \frac{1}{3}e^{3\ln 2}\right) - \left(0 - \frac{1}{3}e^{0}\right)$	M1		Dealing correctly with limits; F(b)–F(0)
	$= 8\ln 2 - \frac{8}{3} + \frac{1}{3}$	A1		Condone awrt 3.21
	$= 8\ln 2 - \frac{7}{3}$	A1	3	**ag** cso (must be exact throughout)
(e)(i)		B1		'Two-branch' graph covering 1st two quadrants only, with one intersection point with x-axis and left of B similar to given left portion of curve.
		B1	2	Reasonable reflection in the x-axis of that part of the given curve that is below the x-axis.
(ii)	$e^{3x} - 8 = 19$	M1		oe
	$x = \frac{1}{3}\ln 27 \quad (= \ln 3)$	A1	2	Accept any **exact** form for this **single** value of x.
	Total		**14**	
	TOTAL		**60**	