

General Certificate of Education
June 2004
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 1**

MBP1

Monday 24 May 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve has equation $y = x^4 - 32x + 7$.

- (a) Find $\frac{dy}{dx}$. (2 marks)
- (b) Find the x -coordinate of the stationary point of the curve. (2 marks)
- (c) State, with a reason, whether the stationary point is a maximum or minimum point. (2 marks)

2 (a) Write each of the following as a power of 2:

- (i) $\sqrt{2}$; (1 mark)
- (ii) 8^x . (1 mark)

(b) Hence solve the equation $8^x \times 2^{x+1} = \sqrt{2}$. (3 marks)

3 The functions f and g are defined for $x > 0$ by

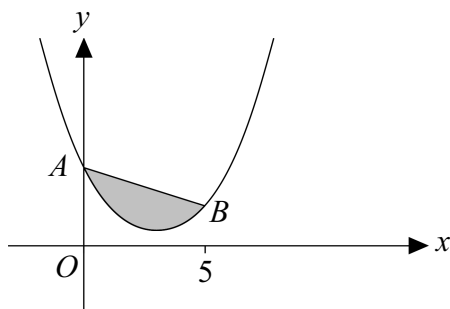
$$f(x) = \frac{5}{x} \quad g(x) = x^2 + 1$$

- (a) Find $fg(x)$. (1 mark)
- (b) When $fg(x) = x$, show that $x^3 + x - 5 = 0$. (2 marks)
- (c) Show that the equation $x^3 + x - 5 = 0$ has a root which lies between 1.5 and 1.6. (2 marks)

4 The points A and B have coordinates $(5, 3)$ and $(9, 9)$ respectively.
The line AC has equation $2x - 3y = 1$.

- (a) (i) Find the gradient of AB . (1 mark)
- (ii) Determine whether the lines AC and AB are perpendicular. (3 marks)
- (b) (i) Given that $2x^2 + 3y = 11$
and $2x - 3y = 1$
show that $x^2 + x - 6 = 0$. (2 marks)
- (ii) Hence find the coordinates of the points of intersection of the curve with equation $2x^2 + 3y = 11$ and the line AC . (4 marks)

- 5 (a) (i) Express $x^2 - 6x + 10$ in the form $(x - p)^2 + q$, where p and q are integers. (2 marks)
- (ii) Describe in detail the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 6x + 10$. (3 marks)
- (b) Prove that the quadratic equation $x^2 - 6x + 10 = 0$ has no real solutions. (3 marks)
- (c) (i) Find $\int (x^2 - 6x + 10) dx$. (2 marks)
- (ii) Hence find $\int_0^5 (x^2 - 6x + 10) dx$. (2 marks)
- (iii) The curve with equation $y = x^2 - 6x + 10$ is sketched below.



The curve crosses the y -axis at the point A , and the x -coordinate of the point B on the curve is 5.

The region bounded by the curve and the line AB , shaded in the diagram, is R . Determine the area of R . (3 marks)

- (d) The points P and Q lie on the curve with equation $y = x^2 - 6x + 10$. The x -coordinate of P is 1 and the x -coordinate of Q is $1 + h$.
- (i) Show that the gradient of the chord PQ is $h - 4$. (3 marks)
- (ii) Deduce the value of the gradient of the curve at the point P . (1 mark)

6 (a) Find the value of $\sum_{r=1}^{29} r^2$. (2 marks)

- (b) (i) The first two terms of an arithmetic series are 3 and 7 respectively.

Write down the r th term of the series, giving your answer in its simplest form.

(3 marks)

- (ii) Express the sum of the following arithmetic series in sigma notation

$$3 + 7 + 11 + \dots + 799$$

(You are not required to evaluate this sum.)

(2 marks)

7 (a) Solve the quadratic equation $2y^2 - 3y - 2 = 0$. (2 marks)

- (b) (i) Show that the equation

$$3 \tan x + 2 \cos x = 0$$

can be written in the form

$$3 \sin x + 2 \cos^2 x = 0 \quad (1 \text{ mark})$$

- (ii) Show further that the equation

$$3 \sin x + 2 \cos^2 x = 0$$

can be written in the form

$$2 \sin^2 x - 3 \sin x - 2 = 0 \quad (2 \text{ marks})$$

- (c) Use the results from parts (a) and (b) to find all the values of x in the interval $0^\circ \leq x \leq 360^\circ$ for which

$$3 \tan x + 2 \cos x = 0 \quad (3 \text{ marks})$$

END OF QUESTIONS