GCE 2004
June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
MBM5

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

Mathematics and Statistics B Mechanics 5 MBM5 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1	R is (1, 5, 5)	M1		
	$\overrightarrow{PR} = \mathbf{r} - \mathbf{p} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$	A1		
	Moment is $(\mathbf{r} - \mathbf{p}) \times \mathbf{F} = \begin{vmatrix} i & j & k \\ -2 & 1 & 4 \\ 7 & -5 & 2 \end{vmatrix}$	M1 A1		
	$= 22\mathbf{i} + 32\mathbf{j} + 3\mathbf{k}$	A1	5	M2 A2 for $-[22\mathbf{i} + 32\mathbf{j} + 3\mathbf{k}]$
	Total		**5**	
2(a)	$I = \int F \, \mathrm{d}t$	M1		
	$= \int_0^{0.2} 30t(0.2 - t) \, \mathrm{d}t$			
	$= \left[3t^2 - 10t^3\right]_0^{0.2}$	M1A1		
	$= 0.12 - 0.08$			
	$= 0.04$	A1	4	
(b)	Using impulse = change in momentum			
	$0.04 = 0.005\,(6 + v)$	M1		
		A1√		
		B1		for 0.005
	$= 0.03 + 0.005v$			
	Speed is 2 ms^{-1}	A1√	4	ft dep on M2 in (a)
	Total		**8**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	Using transverse component of acceleration is $r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$	B1		
	$ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta$	M1		
	$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g\sin\theta}{l}$			
	For small angles, $\sin\theta \approx \theta$	B1		
	$\therefore \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g\theta}{l}$	A1	4	**sc** 3 if lost '–' sign
(b)(i)	$A = \frac{\pi}{20}$	B1		
	$\omega = \frac{1}{2}\sqrt{g}$	B1		
	$\alpha = 0$	B1	3	
(ii)	When change in θ is $\frac{3\pi}{40}$,			
	$\theta = -\frac{\pi}{40}$,	M1 A1		
	$-\frac{1}{2} = \cos\frac{1}{2}\sqrt{g}\,t$	B1		For $-\frac{1}{2}$
	$\frac{1}{2}\sqrt{g}t = \frac{2\pi}{3}$	M1		
	$t = \frac{4\pi}{3\sqrt{g}}$	A1	5	
	Total		**12**	
4	$m = M + \lambda t$	B1		
	Initial $m \to v$			
	Final $m + \delta m \to v + \delta v$			
	Conservation of linear momentum	M1A1		
	$mv = (m + \delta m)(v + \delta v)$			
	$mv = mv + v\delta m + m\delta v$ (to first order of δ terms)			
	$0 = m\delta v + v\delta m$			
	$\therefore (M + \lambda t)\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t} = 0$	M1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = \lambda$	B1		
	$\Rightarrow \therefore (M + \lambda t)\frac{\mathrm{d}v}{\mathrm{d}t} + v\lambda = 0$	A1	6	
	Total		**6**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	Distance perpendicular to slope: $S = V\sin 20\, t - \frac{1}{2} g \cos 20\, t^2$ Strikes slope when $s = 0$ $t = \frac{2V \sin 20}{g \cos 20}$ [t = 0 not required] Velocity perpendicular to slope $V_{\text{perp}} = V \sin 20 - g \cos 20\, t$ $= V \sin 20 - g \cos 20 \frac{2V \sin 20}{g \cos 20}$ $= -V \sin 20$ $[= -0.342V]$ Velocity along slope $V_{\text{along}} = V \cos 20 + g \sin 20\, t$ $= V \cos 20 + g \sin 20 \frac{2V \sin 20}{g \cos 20}$ $= \frac{V}{\cos 20}(\cos^2 20 + 2\sin^2 20)$ $= 1.19V$	M1 A1 A1 M1 A1 M1 A1 M1 A1	9	Could be stated M1 A1 Accept $V\sin 20$
(b)	After rebounding from plane, velocity along the plane is $1.19V$ Velocity perpendicular to the plane is $\frac{1}{3} \times 0.342V = 0.114V$ $\therefore$ Angle direction makes with the plane is $\tan^{-1} \frac{0.114}{1.19}$ $= 5.48°$	B1 M1A1 M1 A1	5	5.4785.. accept 5.47
	Total		**14**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	CF $\ddot{x}+\dot{x}=0$			
	$x = Ae^{nt}$ $\quad n^2+n=0$	M1		
	$n = 0, -1$			
	$x = A + Be^{-t}$	A1		
		A1		
	PI $\quad x = C\cos 2t + D\sin 2t$	M1		Need both terms
	$-4C\cos 2t - 4D\sin 2t - 2C\sin 2t + 2D\cos 2t = k\sin 2t$			
	$-4C + 2D = 0$ and $-4D - 2C = k$			
	$C = -\frac{k}{10}$; $D = -\frac{k}{5}$	A1		A1 if at least one correct
	$x = A + Be^{-t} - \frac{k}{10}\cos 2t - \frac{k}{5}\sin 2t$	A1√		ft dep on M2 above
	$t = 0, x = a \quad a = A + B - \frac{k}{10}$	A1√		ft dep on M2 above
	$\dot{x} = -Be^{-t} + \frac{k}{5}\sin 2t - \frac{2k}{5}\cos 2t$	M1√		
	$t = 0,\ \dot{x} = 0;\quad 0 = -B - \frac{2k}{5}\ \ B = -\frac{2k}{5}$	A1		
	$A = a + \frac{k}{10} - B$			
	$A = a + \frac{k}{2}$	A1		
	$x = a + \frac{k}{2} - \frac{2k}{5}e^{-t} - \frac{k}{10}\cos 2t - \frac{k}{5}\sin 2t$	A1	11	
b	If e^{-t} term may be ignored,			
	range of $\frac{k}{10}\cos 2t + \frac{2k}{10}\sin 2t$			
	using $a\cos\theta + b\sin\theta = R\cos(\theta - \alpha)$	M1		
	is $\pm\frac{k\sqrt{5}}{10}$	M1		M1 [Max/min]
		A1		
	$\therefore$ values are	A1√	4	ft dep on all M gained in (a)
	$a + \frac{k}{2} - \frac{k\sqrt{5}}{10}$ and $a + \frac{k}{2} + \frac{k\sqrt{5}}{10}$			if differentiation used, **sc** 2 for either
	These are $a + 0.276\,k$ and $a + 0.7236k$			
	Total		**15**	
	TOTAL		**60**	