General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 2

MBM2

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A possible model for the acceleration, $a\,\mathrm{m\,s^{-2}}$, of a particle at time t seconds is

$$a = 8 - ht$$

where h is a positive constant.

(a) The acceleration is zero when $t = 4$.

(i) Find h. *(1 mark)*

(ii) Write down an expression for a in terms of t. *(1 mark)*

(b) The velocity of the particle is $2\,\mathrm{m\,s^{-1}}$ when $t = 4$. Find the velocity of the particle at time t. *(4 marks)*

2 The position vector, $\mathbf{r}$, of a particle at time t is given by

$$\mathbf{r} = 4\sin t\mathbf{i} + 4\cos t\mathbf{j} + 6t\mathbf{k}$$

The horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular and the unit vector $\mathbf{k}$ is vertical.

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(c) Show that the magnitude of the acceleration of the particle is 4. *(3 marks)*

(d) Show that the speed of the particle is constant. *(3 marks)*

3 An elastic rope has natural length 4 metres and modulus of elasticity 80 N. A particle, of mass 2 kg, is attached to one end of the rope, and the other end is fixed at the point A. The particle is released from rest at A and falls vertically.

(a) When the rope just becomes taut, find:

(i) the kinetic energy of the particle; *(2 marks)*

(ii) the speed of the particle. *(3 marks)*

(b) (i) The maximum extension of the rope during the motion is x metres. Show that x satisfies the equation

$$10x^2 - 19.6x - 78.4 = 0 \qquad \textit{(4 marks)}$$

(ii) Hence find the maximum length of the rope. *(3 marks)*

(c) State clearly **one** important assumption that you have made. *(1 mark)*

4 A car, of mass 1200 kg, is travelling up a slope at a constant speed of $20\ \text{m s}^{-1}$. The slope is at an angle of 6° to the horizontal. A resistance force of magnitude 420 N also acts on the car when travelling at this speed. In this situation, the power output of the car is a maximum.

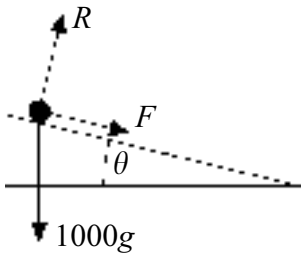
(a) Show that the maximum power output of the car is 33 000 W to three significant figures. *(4 marks)*

(b) The resistance force acting on the car has magnitude kv newtons, where k is a constant and $v\ \text{m s}^{-1}$ is its speed. Find k. *(2 marks)*

(c) Find the maximum constant speed of the car on a horizontal road. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

5 A car, of mass 1000 kg, travels on a banked track at a constant speed of $10\,\text{m s}^{-1}$. The path of the car is a horizontal circle of radius 40 metres. The angle between the track and the horizontal is θ. The diagram shows the three forces acting on the car as it moves round the track, where R is the normal reaction and F is the friction. The car is modelled as a particle.



The forces all act in a vertical plane that contains the centre of the circle.

(a) The angle θ is such that $F = 0$.

(i) Show that $R = \dfrac{9800}{\cos\theta}$. *(2 marks)*

(ii) Find θ. *(5 marks)*

(b) The angle θ is reduced to 3°. The speed of the car and the radius of its circular path are unchanged. Find F. *(6 marks)*

6 A **hollow** cone is formed by rotating the line with equation $y = \dfrac{x}{5}$, for $0 \leqslant x \leqslant 5$, through 360° around the x-axis.

Use integration to show that the centre of mass of the cone is at a distance $\frac{10}{3}$ from the vertex of the cone. *(5 marks)*

7 A particle attached to a spring moves with simple harmonic motion. The particle moves between the points A and B, which are 0.1 m apart. When the particle is 0.01 m from A its speed is $0.6\,\text{m s}^{-1}$.

(a) Show that the period of the motion is $\dfrac{\pi}{10}$ seconds. *(6 marks)*

(b) Find the speed of the particle when it is at the midpoint of AB. *(2 marks)*

(c) Find the magnitude of the maximum acceleration of the particle. *(2 marks)*

8 A sphere of mass m kg is projected vertically from ground level at a speed of $20\,\mathrm{m\,s^{-1}}$. As it moves it experiences a resistance force of magnitude mkv newtons, where k is a constant and $v\,\mathrm{m\,s^{-1}}$ is the speed of the particle when it is at a height of x metres above ground level.

(a) Show that while the sphere is moving upwards

$$\int \frac{v}{g+kv}\,\mathrm{d}v = -x + c$$

where c is a constant. *(4 marks)*

(b) Using the identity

$$\frac{v}{g+kv} \equiv \frac{1}{k} - \frac{g}{k(g+kv)}$$

show that during the upward motion

$$x = \frac{20-v}{k} + \frac{g}{k^2}\ln\left(\frac{kv+g}{g+20k}\right)$$ *(7 marks)*

(c) Find the maximum height of the sphere in terms of g and k. *(2 marks)*

END OF QUESTIONS

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