General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBD2**
Unit Discrete 2

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Question 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

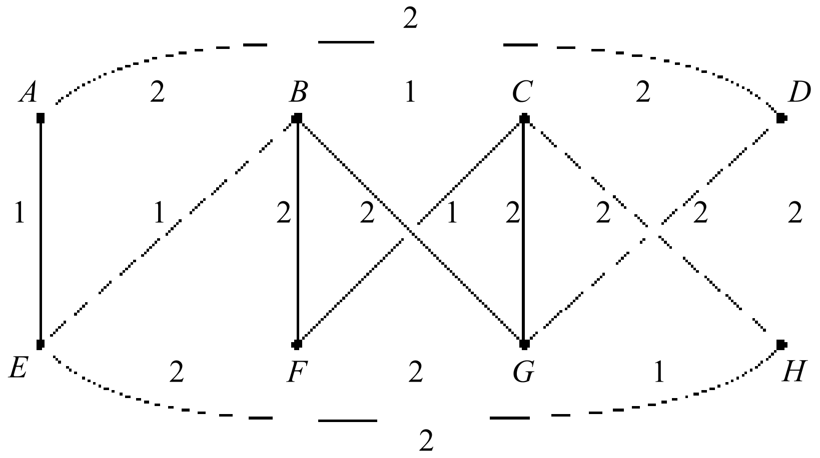
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The network below shows the phone lines connecting eight offices *A*–*H*. The cost per minute for a phone call between two offices is either 1p or 2p as shown on the network.



(a) For the transfer of information, the offices wish to keep a cycle of eight phone lines open. In this way one office will be linked to another, which will be linked to another, and so on until the eighth one is then linked back to the first.

(i) Use a nearest neighbour approach, starting at *A*, to find one suitable cycle of phone lines to keep open. *(3 marks)*

(ii) Explain why the cycle which you found in part (i) is in fact the cheapest possible. *(1 mark)*

(b) The phone company wishes to check all the lines shown in the above network by starting at *A*, passing a message along one line, and then the receiver of the message will pass it along another line, and so on until the message is eventually received back at *A*, having used each of the lines at least once.

(i) Use the Chinese postperson algorithm to find which lines will be used twice when this checking is done in the most economical way. *(4 marks)*

(ii) The condition that the message ends back at *A* is dropped, and the company merely requires a route which uses each line at least once. In the new most economical route, which line, or lines, will be used more than once, and at which office will the message end? *(2 marks)*

2 (a) Find the general solution of the recurrence relation

$$u_n - 5u_{n-1} + 6u_{n-2} = 0$$ *(4 marks)*

(b) Hence find the general solution of the recurrence relation

$$u_n - 5u_{n-1} + 6u_{n-2} = 1$$ *(3 marks)*

3 (a) A linear binary code A has codewords

000000
010111
101011
111100

(i) Calculate the Hamming distance of this code. *(2 marks)*

(ii) State the number of errors in a word which the code can simultaneously detect and correct. *(1 mark)*

(b) A linear binary code B has parity-check matrix

$$\begin{bmatrix} 001100 \\ 111000 \\ 000111 \end{bmatrix}$$

(i) Find two codewords of the code. *(3 marks)*

(ii) A message is received as 110011011001. Use the parity-check matrix to correct this message to the nearest possible one. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

4 When using the simplex method to solve a particular linear programming problem involving three variables, x, y and z the initial tableau is as shown below:

P	x	y	z	s	t	u	
1	−3	−2	−2	0	0	0	0
0	1	1	1	1	0	0	80
0	2	2	1	0	1	0	150
0	2	3	3	0	0	1	180

(a) Express the objective function, P, in terms of x, y and z. *(1 mark)*

(b) (i) Apply one iteration of the simplex method by increasing x. *(5 marks)*

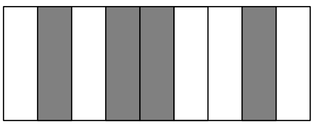
(ii) Explain how you know that the optimal point has not been reached. *(1 mark)*

(c) Perform a second iteration of the simplex method. *(4 marks)*

(d) State the maximum possible value of the objective function and the values of x, y and z for which this maximum is reached. *(2 marks)*

(e) Write down an inequality in x, y and z which is part of the original problem and which still has slack at the optimal point. *(2 marks)*

5 A bookshop is introducing a bar-code system for its books. Each title will have its own bar-code consisting of 9 strips, each of which can be black or white. One example is shown below.



(a) How many different bar-codes are there in this system? *(2 marks)*

(b) The bookshop then decides that each bar-code should have an even number of black strips.

(i) Explain why 256 bar-codes are now possible. *(2 marks)*

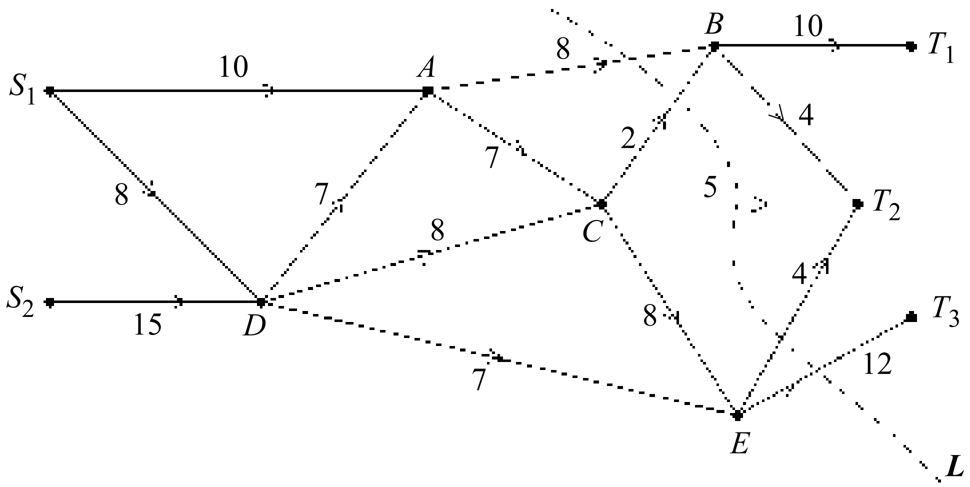
(ii) What advantage does the new system have? *(2 marks)*

(c) The bookshop stocks about 1500 titles and so it is necessary to increase the number of strips from 9, but with each bar-code still having an even number of black strips. Suggest a suitable number of strips. *(2 marks)*

(d) The bar-codes can be passed through the scanner either from left to right or from right to left. Identify a flaw in the bookshop's system and suggest a suitable modification to it. *(3 marks)*

6 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

Two food factories S_1 and S_2 supply three major supermarkets T_1, T_2 and T_3. The various routes from factories to supermarkets are shown in the network below, where the number on each arc represents the daily maximum number of lorry-loads which can use that route.



(a) Add a supersource S and supersink T to **Figure 1**, together with appropriate arcs and capacities, in order to create an equivalent network with a single source and single sink. *(2 marks)*

(b) (i) Calculate the value of the cut shown by the broken line ***L*** above. *(1 mark)*

(ii) Find a cut of lower value than the one given in part (i). *(2 marks)*

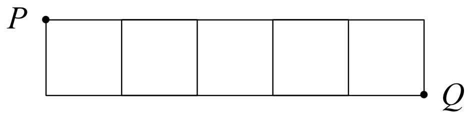
(c) In your adapted network in **Figure 1**, start from a zero flow and use flow-augmenting paths to find a flow of value 30 from S to T. *(6 marks)*

(d) State how you can be sure that the flow found in part (c) is a maximum flow. *(2 marks)*

(e) Show that the three supermarkets cannot each be supplied with 10 lorry-loads in a day. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

7 (a) A grid of footpaths around square fields is as shown, where each of the 5 fields is 1 km by 1 km.

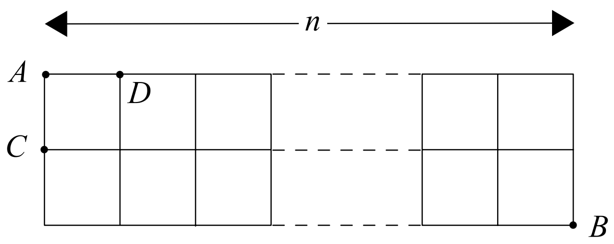


It is possible to walk from P to Q along the paths using various routes of length 6 km.

(i) Show that there are 6 possible routes. *(2 marks)*

(ii) How many routes of minimum length would there be if the grid consisted of n fields rather than 5? *(1 mark)*

(b) Another grid of footpaths around $2n$ fields is as shown below, where each of the fields is 1 km by 1 km.



It is possible to walk from A to B along paths using various routes each of length $(n+2)$ km. The number of such routes is R_n.

(i) Explain why $(n+1)$ of the R_n routes from A to B go through the point C. *(1 mark)*

(ii) Explain why R_{n-1} of the R_n routes from A to B go through the point D. *(1 mark)*

(iii) Deduce that R_n satisfies the recurrence relation

$$R_1 = 3, \qquad R_n = R_{n-1} + n + 1, \qquad n > 1$$ *(3 marks)*

(iv) Deduce that

$$R_n = 1 + 2 + 3 + \ldots + n + (n+1)$$ *(3 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

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Insert for use in Question 6.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

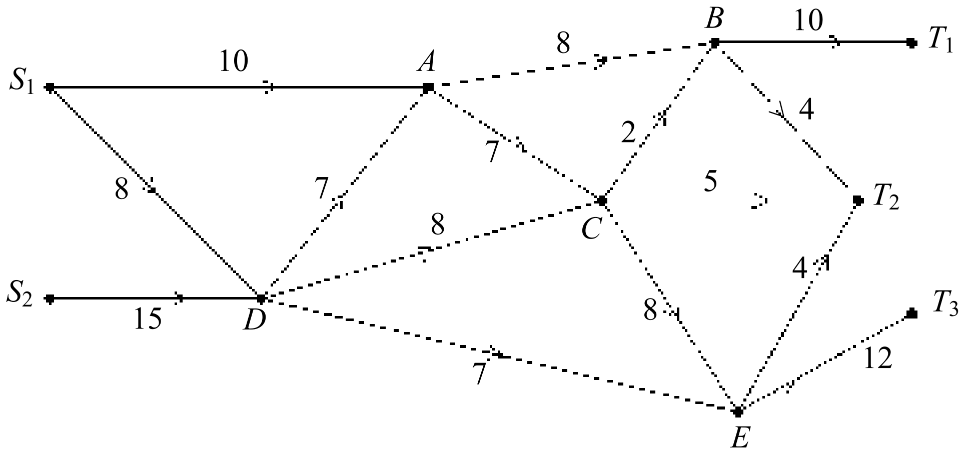

Figure 1

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