AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme
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GCE

Mathematics & Statistics B

Unit MBS7

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Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question Number and Part	Solution	Marks	Total	Comments
1	$n = 25$ and $\sum (x - \bar{x})^2 = 4.08$			
(a)	$X \sim$ normal	B1		oe
	CI for σ^2 is: $\dfrac{\sum (x-\bar{x})^2}{\chi^2(\mathrm{U})}$ to $\dfrac{\sum (x-\bar{x})^2}{\chi^2(\mathrm{L})}$	M1		use of; oe
	Degrees of freedom, $\nu = 25 - 1 = 24$	B1		cao
	95% $\Rightarrow$ 0.025 and 0.975, so values are: 12.401 and 39.364	B1		both; awrt 12.4 and 39.4
	CI for σ^2 is thus : $\dfrac{4.08}{39.364}$ to $\dfrac{4.08}{12.401}$	A1√		ft on χ^2 and equivalent to 4.08
	= (0.104, 0.329)	A1	6	awrt
(b)	$0.25^2 = 0.0625 < 0.104$ $0.25 < \sqrt{0.104} = 0.32$	B1√		ft on CI
	Thus evidence that $\sigma > 5$	B1√	2	ft on CI
	Total		**8**	
2	$D \sim \mathrm{Exp}(125)$			
(a)	$\mathrm{P}(D < 100) = \left[-\mathrm{e}^{-\frac{d}{125}} \right]_0^{100} = 1 - \mathrm{e}^{-\frac{100}{125}}$	M1		use of PDF or DF
	$= 1 - \mathrm{e}^{-0.8} = 0.550$ to 0.551	A1	2	awfw; accept 0.55
(b)	$\mathrm{P}(100 < D < 300) = \mathrm{P}(D < 300) - (\mathrm{a})$	M1		use of; oe
	$(1 - \mathrm{e}^{-2.4}) - (1 - \mathrm{e}^{-0.8}) = \mathrm{e}^{-0.8} - \mathrm{e}^{-2.4}$ $= 0.909 - 0.551 = 0.449 - 0.091$	A1√		correct expression; oe ft on (a)
	$= 0.358$ to 0.359	A1	3	awfw
	Total		**5**	

Question Number and Part	Solution	Marks	Total	Comments
3(a)	$\hat{\beta} = \dfrac{-1587.6}{441} = -3.6$	B1		cao
	$s^2 = \dfrac{1}{n-2}\left(S_{yy} - \dfrac{S_{xy}^2}{S_{xx}}\right) =$	M1		use of; oe
	$\dfrac{1}{25}\left(6940.36 - \dfrac{(-1587.6)^2}{441}\right) = \dfrac{1225}{25}$ $= 49$	A1	3	awrt
(b)	H_0: $\beta = -3$ H_1: $\beta \neq -3$	B1		both
	SL $\alpha = 0.10$ DF $\nu = 27 - 2 = 25$ CV $t = \pm 1.708$	B1 B1		cao awrt 1.71; ignore sign
	$t = \dfrac{\hat{\beta} - \beta_0}{\sqrt{\dfrac{s^2}{S_{xx}}}}$	M1		use of; accept no β_0
	$t = \dfrac{-3.6 - (-3)}{\sqrt{\dfrac{49}{441}}} = -1.80$	A1		awrt; accept –1.8
	Thus evidence, at 10% level, to reject the claim that $\beta = -3$	A1√	6	ft on t and CV, providing consistent signs
	Total		**9**	

Question Number and Part	Solution	Marks	Total	Comments
4	H_0: number is constant H_1: number is not constant	B1		at least H_0
	SL $\alpha = 0.10$ DF $\nu = 7 - 1 = 6$ CV $\chi^2 = 10.645$	 B1 B1		 cao awfw 10.6 to 10.7
	Mean per hour = $\frac{\sum \text{calls}}{7}$ =	M1		use of
	$\frac{931}{7} = 133$	A1		cao
	$\chi^2 = \sum \frac{(O-E)^2}{E}$ =	M1		use of
	$\frac{1}{133}\sum (O-133)^2 = 5.73$	A1		awfw 5.72 to 5.74
	Thus insufficient evidence, at 10% level, to suggest that number per hour is not constant	A1√	8	ft on χ^2 and upper CV
	Total		**8**	

Question Number and Part	Solution	Marks	Total	Comments
5	$X \sim \text{N}(220, 20^2)$ $Y \sim \text{N}(175, 40^2)$			
(a)	$T = X + Y$ has:			
	mean = 395	B1		cao
	and			
	variance = 2000	B1		cao; accept sd = 44.7 awrt
	$\text{P}(T < 300) =$			
	$\text{P}\left(Z < \frac{300 - 395}{\sqrt{2000}}\right) =$	M1		standardising 300 using their μ and their σ
	$\text{P}(Z < -2.12) = \Phi(-2.12) = 1 - \Phi(2.12)$	m1		attempted area change
	$= 0.0165$ to 0.0170	A1	5	awfw; accept 0.017
(b)	$D = X - Y$ has:	M1		use of difference
	mean $= \pm 45$	A1		cao; ignore sign
	variance = 2000			both mean and variance cao; accept sd = 44.7 awrt
	$\text{P}(D > 0)$			
	$= \text{P}\left(Z > \frac{0 - 45}{\sqrt{2000}}\right)$	M1		standardising 0 using their μ and their σ
	$= \text{P}(Z > -1.01) = \Phi(1.01)$			
	$= 0.841$ to 0.844	A1	4	awfw
	Total		**9**	

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$n = 40$			
	H_0: $p = 0.10$ (10%) H_1: $p > 0.10$ (10%)	B1		both; can be scored in (b)
	$P(X \geq 7 \mid 40, 0.1)$	M1		attempt at using B(40, 0.1) or Po(4)
	$= 1 - P(X \leq 6)$	A1		1 – and ≤ 6
	$= 1 - 0.9005 = 0.10$ (> 0.05)	A1		awrt; accept 10% (≥ 8 (CR) gives 0.0419)
	Thus insufficient evidence, at 5% level, to support buyer's suspicion	A1√	5	ft on p-value and 0.05 (5%) or on 7 and CV (8)
(b)	$n = 400$			
	Normal approximation with	M1		use of
	mean (μ) = 40 and variance (σ^2) = 36	A1		cao; both
	SL $\alpha = 0.05$ CV $z = 1.6449$	B1		awfw 1.64 to 1.65
	$z = \dfrac{x - \mu}{\sqrt{\sigma^2}}$	M1		standardising (51.5, 52, 52.5) using their μ and their σ
	$z = \dfrac{(51.5 \text{ or } 52) - 40}{6} =$			1.91 ⇒ p-value of 0.028 2.00 ⇒ p-value of 0.023 (binomial ⇒ 0.031)
	1.91 to 2.00	A1		awfw; accept 2
	Thus sufficient evidence, at 5% level, to support buyer's suspicion	A1√	6	ft on z and CV or on p-value and 0.05 (5%)
	Total		**11**	

Question Number and Part	Solution	Marks	Total	Comments
7	V n $\bar{x}$ s^2 P 11 201 124 Q 16 188 134			allow use of suffices x/1/P and y/2/Q throughout question
(a)	$s_p^2 = \dfrac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$	M1		allow misuse of $(s^2)^2$
	Thus $s_p^2 = \dfrac{10 \times 124 + 15 \times 134}{25}$	A1	2	cao
	$= \dfrac{3250}{25}$ (= 130)			**ag**
(b)	H_0: $\mu_x = \mu_y$	B1		cao; oe
	H_1: $\mu_x > \mu_y$	B1		cao; oe
	SL $\alpha = 0.01$			
	DF $\nu = 11 + 16 - 2 = 25$	B1		cao
	CV $t = 2.485$	B1		awfw 2.48 to 2.49
	$t = \dfrac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{\sqrt{s_p^2\left(\dfrac{1}{n_x} + \dfrac{1}{n_y}\right)}}$	M1		use of; accept $(\mu_x - \mu_y) = 0$ not z
	Thus $t = \dfrac{201 - 188}{\sqrt{130\left(\dfrac{1}{11} + \dfrac{1}{16}\right)}}$	A1√		substitution; ft on s^2 only
	= 2.91	A1		awrt
	Thus evidence, at 1% level, that pears of Variety P weigh, on average, more than pears of Variety Q (grower's suspicion)	A1√	8	ft on t/z and CV
	Total		**10**	
	TOTAL		**60**	