General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Pure 5

MBP5

ASSESSMENT and QUALIFICATIONS ALLIANCE

Monday 19 January 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Use the trapezium rule with five ordinates (four strips) to find an approximation to

$$\int_1^3 \frac{1}{x^3+3}\,\mathrm{d}x$$

giving your answer to 3 significant figures. *(4 marks)*

2 Given that $y^3 + 3y = x^3$, use implicit differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2}{y^2+1}$$ *(3 marks)*

3 (a) Obtain the first three terms of the binomial expansion of $(1+4x^2)^{\frac{1}{2}}$ in ascending powers of x. *(3 marks)*

(b) State the range of values of x for which the full expansion is valid. *(2 marks)*

(c) By integrating the three terms in your expansion, find an approximate value for

$$\int_0^{\frac{1}{4}} (1+4x^2)^{\frac{1}{2}}\mathrm{d}x$$ *(3 marks)*

4 (a) Express $12\sin 2x + 16\cos 2x$ in the form $R\sin(2x+\alpha)$, where R is a positive constant and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 significant figures. *(3 marks)*

(b) A curve has equation $y = 11x^2 - 3\sin 2x - 4\cos 2x$.

(i) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(3 marks)*

(ii) Show that the curve does not have any points of inflection. *(2 marks)*

5 A curve has equation $y = \dfrac{x^2}{x^2 + 3x + 3}$.

(a) Write down the equation of the horizontal asymptote to the curve. *(1 mark)*

(b) (i) Prove that, for all real values of x, y satisfies the inequality $0 \leqslant y \leqslant 4$. *(6 marks)*

(ii) Hence find the coordinates of the turning points on the curve. *(3 marks)*

(c) Given that there are no vertical asymptotes, sketch the curve. *(3 marks)*

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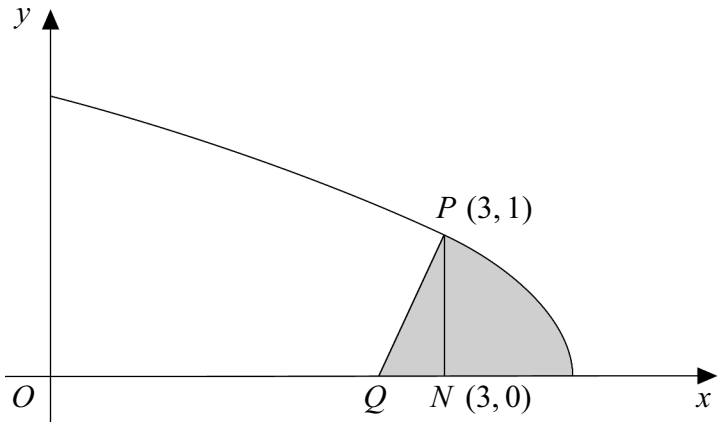

The diagram shows the curve C which is defined parametrically by

$$x = 4\sin^2 t, \qquad y = 2\cos t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The point $P\ (3, 1)$ lies on the curve C and the foot of the perpendicular from P to the x-axis is $N\ (3, 0)$. The normal to the curve C at P intersects the x-axis at the point Q.

(a) (i) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. *(3 marks)*

(ii) Find the value of t at the point P. *(1 mark)*

(iii) Show that the equation of the normal PQ is $y = 2x - 5$. *(3 marks)*

(b) (i) Show that the area of the region bounded by the curve C, the line PN and the x-axis is given by $16\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \, \mathrm{d}t$. *(3 marks)*

(ii) Using the substitution $u = \cos t$, or otherwise, evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \, \mathrm{d}t$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve C, the normal PQ and the x-axis. *(2 marks)*

7 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

The plane Π has equation $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 5$.

(a) Find the value of $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$. *(1 mark)*

(b) Find the position vector of the point of intersection of l and Π. *(3 marks)*

(c) (i) Show that the angle between the vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ is

$$\cos^{-1}\left(\frac{4\sqrt{70}}{35}\right)$$ *(3 marks)*

(ii) Hence find, to the nearest degree, the angle between l and Π. *(2 marks)*

END OF QUESTIONS