General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBP4**
(SPECIFICATION B)
Unit Pure 4

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The population, P, of insects in a colony is given by $P = Ae^{kt}$, where A and k are constants and the time t is measured in months.

(a) Given that $P = 500$ when $t = 0$ and that $P = 750$ when $t = 10$, find the value of k. *(3 marks)*

(b) Find the value of t when $P = 1500$, giving your answer to 3 significant figures. *(2 marks)*

2 A model plane moves so that its height, y metres, above horizontal ground is given by

$$y = \frac{8x}{x^3 + 1}, \qquad x \geqslant 0$$

when its horizontal distance from the take-off point on the ground is x metres.

(a) Find the value of $\frac{dy}{dx}$ when $x = 1$. *(3 marks)*

(b) (i) Find the rate of change of y in $m\,s^{-1}$ when $x = 1$ and x is increasing at a rate of $0.8\,m\,s^{-1}$. *(2 marks)*

(ii) Interpret the sign in your answer to part (i). *(1 mark)*

3 The polynomial $p(x)$ is given by

$$p(x) = (x+3)(x-2)(x-4)$$

(a) Find the remainder when $p(x)$ is divided by $(x+1)$. *(2 marks)*

(b) (i) Express $\frac{70}{(x+3)(x-2)(x-4)}$ in the form $\frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{x-4}$. *(3 marks)*

(ii) Hence, prove that $\int_5^6 \frac{70}{(x+3)(x-2)(x-4)}\,dx = N\ln 3 - M\ln 2$, where N and M are positive integers. *(5 marks)*

4 A circle has equation $x^2 + y^2 - 10x - 6y + \frac{111}{4} = 0$.

(a) (i) Find the coordinates of the centre, C. *(2 marks)*

(ii) Find the radius of the circle. *(2 marks)*

(b) The line l_1 has equation $3x - 4y - 16 = 0$.

(i) Find the distance from C to l_1. *(2 marks)*

(ii) Hence determine whether the line l_1 intersects the circle. *(1 mark)*

(iii) The line l_2 has equation $y = 2x + 5$.

Show that the acute angle between l_1 and l_2 is $\tan^{-1}\frac{1}{2}$. *(3 marks)*

5 A sequence is defined by

$$x_{n+1} = \sqrt{(x_n + 12)}, \qquad x_1 = 2$$

(a) Find the values of x_2, x_3, and x_4, giving your answers to 3 decimal places. *(3 marks)*

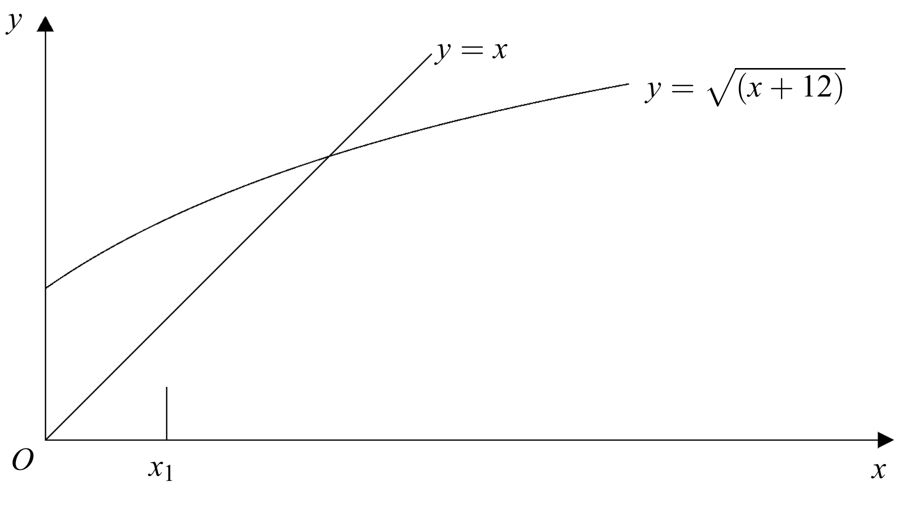
(b) Given that the limit of the sequence is L:

(i) show that L must satisfy the equation $L^2 - L - 12 = 0$; *(2 marks)*

(ii) find the value of L. *(2 marks)*

(c) The graphs of $y = \sqrt{(x + 12)}$ and $y = x$ are sketched below.

On a copy of the sketch, draw a cobweb or staircase diagram to show how convergence takes place.



(2 marks)

Turn over ▶

6 (a) Use the identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

to show that the equation

$$\cos\left(x + \frac{5\pi}{6}\right) = \sin x$$

can be written as

$$\cos x + \sqrt{3} \sin x = 0$$ *(4 marks)*

(b) Hence solve the equation

$$\cos\left(x + \frac{5\pi}{6}\right) = \sin x$$

giving all solutions, in terms of π, in the interval $0 < x < 2\pi$. *(3 marks)*

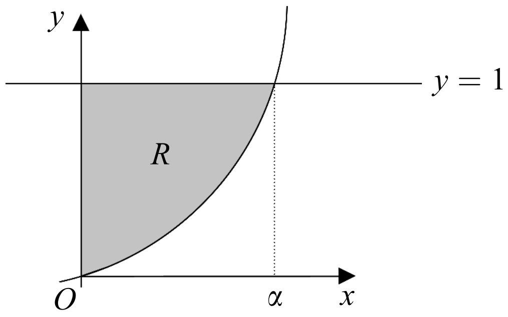
7 A curve is defined for $0 \leqslant x < \frac{\pi}{4}$ by the equation $y = \tan 2x$.

(a) (i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Hence, find the equation of the tangent to the curve at the point $\left(\frac{\pi}{6}, \sqrt{3}\right)$. *(2 marks)*

(b) Find $\int (\sec^2 2x - 2\tan 2x)\,dx$. *(3 marks)*

(c) The curve $y = \tan 2x$ intersects the line $y = 1$ when $x = \alpha$, as shown in the diagram.



(i) Find the value of α in terms of π. *(1 mark)*

(ii) The shaded region bounded by the curve, the y-axis and the line $y = 1$ is R.

The region R is rotated through 2π radians about the line $y = 1$.

Show that the volume of the solid of revolution, V, is given by

$$V = \pi \int_0^{\alpha} (\sec^2 2x - 2\tan 2x)\,dx$$ *(3 marks)*

(iii) Prove that $V = \frac{\pi}{2}(1 - \ln 2)$. *(2 marks)*

END OF QUESTIONS